Uses of Short Term-indicators for Forecasting

International Workshop Beijing

Klaus Abberger

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Outline

1 Motivation

2 Forecasting with Time Series Regressions
   - Autoregressive Distributed Lag Models
   - Error Correction Models
   - Mixed Frequency Analysis

3 Turning Points and Composite Indicators
   - Binary response models
   - Markov-Switching Models

4 Forecast Assessment
Outline

1. Motivation
2. Forecasting with Time Series Regressions
3. Turning Points and Composite Indicators
4. Forecast Assessment
<table>
<thead>
<tr>
<th>Month 1</th>
<th>Month 2</th>
<th>Month 3</th>
<th>Month 4</th>
<th>Month 5</th>
<th>Month 6</th>
<th>Month 7</th>
<th>Month 8</th>
<th>Month 9</th>
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<td>Quarter III</td>
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</table>
GDP/National Account

Month 1  Month 2  Month 3  Month 4  Month 5  Month 6  Month 7  Month 8  Month 9
Quarter I  Quarter II  Quarter III
GDP/National Account

Flash Estimate $t+45$

Month 1 Month 2 Month 3 Month 4 Month 5 Month 6 Month 7 Month 8 Month 9

Quarter I Quarter II Quarter III
GDP/National Account

Backcasting

Nowcasting

Forecasting

Forecasting exercise

Flash Estimate

I.

II.

III.

Quarter I

Quarter II

Quarter III

Month 1
Month 2
Month 3
Month 4
Month 5
Month 6
Month 7
Month 8
Month 9

Time
Composite Indicators

- Leading indicators
- Coincident indicators (but publication lead)
Composite Indicators: Uses

- Nowcasting
- Short term forecasting (e.g. next quarter)
- (Backcasting, revision prediction)
The nowcast of GDP has been produced by the dynamic factor model suggested in Giannone et al. (2008). The model has been estimated using 555 indicators related to the Swiss economy that are sub-divided into the following 9 blocks: Purchasing Managers Index in manufacturing supplied by Credit Suisse (9 time series, "PMGR"), consumer price indices (28, "CPI"), labor market indicators (6, "LABOUR"), producer price indices (11, "PPI"), business tendency surveys in manufacturing collected at the KOF Swiss Economic Institute (150, "CHINOGA"), exports and imports (249, "TRADE"), stock market indices (79, "STMKT"), interest rates (20, "INT.RATE"), and exchange rates (3, "EXCH.RATE"). The forecasting performance of the model was investigated in Siliverstovs and Kholodilin (2012) in a pseudo real-time simulation setup. Siliverstovs (2012) investigates the forecasting performance of the model in real time.

Table 1 reports several most recent vintages of real GDP growth (quarterly, seasonally adjusted). Table 2 reports the most recent nowcast for the current quarter and tracks record of past nowcasts and first releases of GDP growth by SECO. Figures 1, 2 and 3 display real-time nowcasts for the years 2010-2011, 2012-2013 and for 2014 together with the first quarter of 2015, respectively. SECO estimates of GDP growth are shown by straight lines (a bold line corresponds to the first release for a particular quarter).

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Target of forecasts

- Which frequency (quarterly growth, annual growth, quarterly growth as an instrument for annual growth where quarterly figures are only partly published)
- Which vintage/publication (first publication, last publication)
### GDP Nowcasting at KOF

<table>
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<tr>
<th>Target</th>
<th>Nowcast</th>
<th>Actual (SECO)(^a)</th>
<th>Target</th>
<th>Nowcast</th>
<th>Actual (SECO)</th>
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<td>2013Q1</td>
<td>0.55 [-0.18, 1.28]</td>
<td>0.56</td>
<td>2014Q1</td>
<td>0.69 [-0.03, 1.41]</td>
<td>0.46</td>
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<td>0.50 [-0.21, 1.21]</td>
<td>0.52</td>
<td>2014Q2</td>
<td>0.56 [-0.15, 1.27]</td>
<td>-0.04 (0.19)(^b)</td>
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<td>03.09.2013</td>
<td>Released on</td>
<td>29.05.2014</td>
<td>02(30).09.2014</td>
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<td>2013Q3</td>
<td>0.44 [-0.27, 1.15]</td>
<td>0.52</td>
<td>2014Q3</td>
<td>0.39 [-0.33, 1.11]</td>
<td>0.63</td>
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<td>02.09.2014</td>
<td>03.12.2014</td>
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<td>2013Q4</td>
<td>0.55 [-0.18, 1.28]</td>
<td>0.16</td>
<td>2014Q4</td>
<td>0.28 [-0.41, 0.97]</td>
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<td>Released on</td>
<td>03.12.2014</td>
<td>03.03.2015</td>
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Macroeconomic model consisting of Error Correction Models and equations to ensure consistency. This model is used for quarterly forecasts of various variables up to two years. Consistency of the model forecasts is crucial.

Nowcasting and short term forecasting with indicator model. These forecasts are used to adjust the model forecasts in the short term.
Composite Indicators

Some specific topics in quantification of composite indicators

- Target/reference series?
  - Which series (e.g. GDP)
  - Which transformation (e.g. annual growth, quarter-on-quarter growth)
  - Which release (first release, release after some revisions)

- Statistical methods (e.g. time series regressions)

- Frequency of indicators and target the same or different?
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   - Autoregressive Distributed Lag Models
   - Error Correction Models
   - Mixed Frequency Analysis

3 Turning Points and Composite Indicators

4 Forecast Assessment
Autoregressive Distributed Lag Model

With $Y_t$ the target variable at time $t$ and $IN_t$ an (composite) indicator at time $t$ one can try to estimate the model

$$Y_t = \alpha + \sum_{i=1}^{m} \beta_i Y_{t-i} + \sum_{j=0}^{n} \gamma_j IN_{t-j} + \epsilon_t,$$

with error term $\epsilon$.

This approach can be used to test whether the coefficients $\gamma$ are significant different from zero. In addition this model can be used to calculate indicator based forecasts.
Error Correction Model

With $Y_t$ the target variable at time $t$ and $X_t$ an independent variable. An Error Correction Model is

$$\Delta_1 Y_t = \alpha + \zeta Z_{t-1} + \sum_{i=1}^{m} \beta_i \Delta_1 Y_{t-i} + \sum_{j=0}^{n} \gamma_j \Delta_1 X_{t-j} + \epsilon_t,$$

with error term $\epsilon$ and

$$Z_t = Y_t - bX_t.$$

$\Delta_1 Y$ will respond negatively to $Z$ since $Z$ is the deviation from the long run equilibrium state ($Z = 0$). $\zeta$ gives the speed of adjustment to the equilibrium.
Error Correction Model

- Estimates short term and long term effects
- Applications to stationary and nonstationary data
- Estimation with OLS
Error Correction Model

- Motivating ECM with cointegrated data
  - Handles nonstationary cointegrated variables
  - Two step estimation procedure
  - Models with only differenced variables may ignore long term effects

- Motivating ECM with stationary data
  - Long and short term effects
  - ECM and ADL are equivalent
Mixed Frequencies

Often time series with different frequency are used for forecasting. So one may want forecast quarterly GDP with a monthly composite indicator. The objective is to forecast a lower-frequency variable, $Y$, sampled at periods denoted by time index $t$. Past realizations of the lower-frequency variable are denoted by the lag operator, $L$. For example, if $Y_t$ is the quarterly GDP, then the GDP one quarter prior would be the first lag of $Y_t$, $LY_t = Y_{t-1}$, two months prior would be $L^2 Y_t = Y_{t-2}$, and so on.
Mixed Frequencies

In addition to lags of $Y$, we are interested in the information content of a higher-frequency variable, $X$, sampled $m$ times between samples of $Y$ (e.g., between $t - 1$ and $t$). $L_{HF}$ denotes the lag operator for the higher-frequency variable. If $X_t$ is the monthly indicator, then $L_{HF} X_t$ denotes the indicator value of the last month of the previous quarter.
Time Aggregation

One solution to the problem of mixed sampling frequencies is to convert higher-frequency data to match the sampling rate of the lower-frequency data. One can for example calculate averages of $X$ between samples of the low frequency variable:

$$
\bar{X}_t = \frac{1}{m} \sum_{k=1}^{m} L_{LH}^k X_t.
$$

With the two variables in the same time domain on can use a regression model for forecasting:

$$
Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{j=1}^{n} \gamma_j L^j \bar{X}_t + \epsilon_t
$$
Time Aggregation

The standard aggregation methods depend on the stock/flow nature of the variables and, typically, it is the average of the high-frequency variables over one low-frequency period for stocks, and the sum for flows.

Taking the latest available value of the higher frequency variable is another option for both stock and flow variables. The underlying assumption is that the information of the previous high-frequency periods is reflected in the latest value, representative of the whole low-frequency period.
Bridge Models

With time aggregation often the problem occurs, that at a specific time point not all of the high-frequency data is already available. The Bridge Model Approach consist of two steps:

1. Forecasting the high-frequency series with time series methods (e.g. ARMA) for all time points which are needed for time aggregation.

2. An ADL model is used at the low frequency to obtain forecasts for $Y$. 
Step Functions

The time aggregation method assumes the slope coefficients on each of the individual observations of $X$ are equal. Alternatively, one could assume that each of the slope coefficients for each $k$ sampling of $X$ are unique. This model, including one lag of the predictor $X (n = 1)$, is

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{k=1}^{m} \gamma_k L^k_{HF} X_t + \epsilon_t.$$ 

The model can be estimated with OLS.
Step Functions

Once the model is extended to multiple lags, the number of parameters could become quite large (especially, when the high frequency data is of higher frequency than monthly). A more general model is

\[ Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{k=1}^{m \cdot n} \gamma_k L_{HF}^k X_t + \epsilon_t \]

which allows for up to \( n \) lower-frequency lags. So the problem with this type of models is, that the number of coefficient can get huge.
MIDAS (Mixed Data Sampling)

The time-averaging model is parsimonious but discards any information about the timing of innovations to higher-frequency data. The step-weighting model preserves the timing information but requires the user to estimate a potentially large number of parameters.
To solve the problem of parameter proliferation while preserving some timing information, MIDAS models can be used:

\[ Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{k=1}^{m} \Phi(k; \theta) L_{HF}^k X_t + \epsilon_t. \]

where the function \( \Phi(k; \theta) \) is a polynomial that determines the weights for temporal aggregation. The weighting function, \( \Phi(k; \theta) \), can have any number of functional forms; the desire here is to achieve flexibility while maintaining parsimony.
Suggestions for $\Phi(k; \theta)$ are often a beta function formulation or an exponential Almond specification. The later is

$$\Phi(k; \theta_1, \theta_2) = \frac{\exp(\theta_1 k + \theta_2 k^2)}{\sum_{j=1}^{m} \exp(\theta_1 j + \theta_2 j^2)}$$

In this case, simple time averaging is obtained when $\theta_1 = \theta_2 = 0$. 
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Turning Points in the Business Cycle

Generally, practitioners in business cycle analysis sometimes assume that economic cycles are constituted by an alternation of two conjunctural phases, namely a phase of high economic activity (or expansion) and a phase of low economic activity (or contraction). These phases can be defined in classical, growth or growth rate cycles. Sometimes also or than two phases are considered.

The objective of parametric models is to provide, at each date $t$, an estimated probability of being in a specific phase.
Binary response models

If there is a reference series and if the phases (dating) of the reference series are available, a binary variable can be defined that takes the value 1 when the economy belongs to one phase and 0 when it belongs to the other phase. This 0–1 variable can be used for logit or probit regressions.
Logistic Regression

Let $Y$ be a binary variable with values 0 and 1 and $X$ a predictor (e.g. a composite indicator), the logistic regression model (logit) is

$$
\logit\left(\frac{\text{prob}(Y_t = 1)}{1 - \text{prob}(Y_t = 1)}\right) = a + bx_t.
$$

The model can be extended to contain lags of $X$ and lags of $Y$. 
Markov Switching

Markov switching models consist to the class of nonlinear time series models. They base on the idea of probability switching between various states (e.g. upswing and downswing). In the following Markov switching autoregressive models are discussed. Markov switching regression models use also explanatory variables.
Markov Switching

Hamilton (1989) considers the Markov switching autoregressive (MSA) model. Here the transition is driven by a two-state Markov chain. A time series $x_t$ follows an MSA model if it satisfies:

$$
x_t = \begin{cases} 
    c_1 + \sum_{i=1}^{p} \phi_{1,i} x_{t-i} + a_{1,t} & \text{if } s_t = 1, \\
    c_2 + \sum_{i=1}^{p} \phi_{2,i} x_{t-i} + a_{2,t} & \text{if } s_t = 2, 
\end{cases}
$$  \hspace{1cm} (2)

where $s_t$ assumes values in $\{1, 2\}$ and is a first-order Markov chain with transition probabilities

$$
P(s_t = 2|s_{t-1} = 1) = w_1, \quad P(s_t = 1|s_{t-1} = 2) = w_2. \quad (3)
$$
Markov Switching

The innovational series $\{a_{1,t}\}$ and $\{a_{2,t}\}$ are sequences of iid random variables with mean zero and finite variance and are independent of each other. A small $w_i$ means that to model tends to stay longer in state $i$. In fact, $1/w_i$ is the expected duration of the process to stay in state $i$. 
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1. Motivation
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Notation

Let \( \{ y_t \}_{t=1}^T \) a time series being forecasted and \( \{ \hat{y}_{it} \}_{t=1}^T \) and \( \{ \hat{y}_{jt} \}_{t=1}^T \) two forecasts. Let the associated forecast errors be \( \{ e_{it} \}_{t=1}^T \) and \( \{ e_{jt} \}_{t=1}^T \).
Measures of prediction error

Mean prediction error:

\[ MPE(\hat{y}_i) = \frac{1}{T} \sum_{t=1}^{T} e_{it} \]

Mean absolute prediction error:

\[ MAPE(\hat{y}_i) = \frac{1}{T} \sum_{t=1}^{T} |e_{it}| \]

Mean squared prediction error:

\[ MSPE(\hat{y}_i) = \frac{1}{T} \sum_{t=1}^{T} (e_{it})^2 \]
## GDP Forecasts for Germany

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<th>EU</th>
<th>SVR</th>
<th>OECD</th>
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## GDP Forecasts for Germany

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Source: Döhrn (2015)
GDP Forecasts for Germany

Evaluation of Business Cycles

Source: Döhrn (2015)
Measures of prediction error

Signal to noise ratio:

\[ SNR(\hat{y}_i) = \frac{MSPE(\hat{y}_i)}{\sigma_y^2} \]
### GDP Forecasts for Germany

<table>
<thead>
<tr>
<th>Component</th>
<th>Mean Squared Error</th>
<th>Noise-to-Signal-Ratio</th>
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<tr>
<td>Private Consumption</td>
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<td>Public Consumption</td>
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<td>GDP</td>
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<td>0.73</td>
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</table>

Source: Döhrn (2015)
Comparision of forecast errors

Theil Coefficient

\[ \text{Theil}_{ij} = \frac{\text{MSPE}(\hat{y}_i)}{\text{MSPE}(\hat{y}_j)} \]

Denote the loss associated with forecast error \( e_t \) be \( L(e_t) \); hence the time-\( t \) quadratic loss would be \( L(e_t) = e_t^2 \). The time-\( t \) loss differential between forecast \( i \) and \( j \) is then

\[ d_{ijt} = L(e_{it}) - L(e_{jt}) \]

and means of the loss differential can be calculated. Typical loss function are squared or absolute loss. But for example also asymmetric loss can be calculated.
Diebold Mariano Test

The Diebold Mariano test is an asymptotic test of the hypothesis that the mean of the loss differential is zero. In practice the test can be calculated by regression of the observed loss differential on an intercept, using heteroscedasticity and autocorrelation robust (HAC) standard errors for testing significance of the intercept.

One can potentially extend the regression to condition on additional variables that may explain the loss differential, thereby moving from an unconditional to a conditional expected loss perspective. For example, comparative predictive performance may differ by stage of the business cycle, in which case one might include a 0-1 business cycle chronology variable in the HAC regression.
Diebold Mariano Test

The common test takes the estimated intercept $\alpha$ and divide it by the HAC estimated standard deviation $s_\alpha$. The test statistic is then

$$DM = \frac{\alpha}{s_\alpha}$$

This statistic is asymptotically standard normal distributed. In small samples (few forecasts) the Null hypothesis is rejected too often, so an small sample correction can be used. With $T$ the number of forecasts and $h$ the forecast horizon the correction is

$$MDM = DM \cdot \sqrt{\frac{N + 1 - 2h + \frac{h(h-1)}{N}}{N}}$$

In addition it is proposed to use the t-distribution with $T - 1$ degrees of freedom to calculate critical values.
Measures of prediction error for a binary variable

When the target variable, \( r_t \), is a binary indicator while the forecast is a probability of a state, \( p_t \), similar techniques can be used as in the case of continuous variables. The quadratic probability score is:

\[
QPS(p) = \frac{1}{T} \sum_{t=1}^{T} 2(p_t - r_t)^2
\]

QPS ranges between \([0, 2]\) with 0 perfect accuracy. A similar loss function that assigns more weight to larger forecast errors is the log probability score:

\[
LPS(p) = \frac{1}{T} \sum_{t=1}^{T} ((1 - r_t) \log(1 - p_t) + r_t \log p_t)
\]

The range of LPS is \([0, \infty]\) with 0 perfect accuracy.
Measures of prediction error for a binary variable

Contingency tables can also be used for a descriptive evaluation of the methodology in the case of binary forecasts and outcomes. They provide a summary of the percentage of correct predictions, missed signals (e.g. no prediction of slowdown when it takes place), and false alarms.