Quarterly National Accounts

Benchmarking Principles

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UN Statistics Division

Discrepancy Between QNA and ANA

Discrepancies occur due to:

- Independent collection of quarterly and annual data sources for the same phenomena
  - Quarterly data based on (smaller) sample surveys using simplified questionnaires
  - Annual data based on censuses/larger sample surveys using more comprehensive questionnaires
  - Annual data based on data from audited business accounts

Discrepancy Between QNA and ANA

Discrepancies occur due to:

- More information, and more detailed information, available annually
- Quarterly estimates based on:
  - Fixed input output coefficients
  - Trend extrapolations
  - Guesstimates

Discrepancy Between QNA and ANA

Discrepancies occur due to:

- Compilation procedures for annual and quarterly national accounts estimates may differ
  - Annual accounts more detailed
  - Annual accounts more complete
  - Use of supply and use tables as compilation tool for the annual accounts
  - Use of simplified methods in the quarterly accounts

Benchmarking

- Benchmarking is a mathematical procedure that makes the information coming from the high frequency series (quarterly) coherent with the low frequency series (annual)
- Objective is to derive a consistent time series that preserves the short-term movements of the quarterly indicator subject to constraint that quarterly sum equals the annual benchmarks.

Benchmarking: Basic Approaches

- Manual reconciliation and revision of independent annual and quarterly data sources, and annual and quarterly estimates

- Mechanical methods
  - Two types
  - Pro-rata distribution – breaks in series (the step problem)
  - Time series method – no breaks in series

Pro Rata Distribution and Step Problem

Distribution presentation

$$X_{q,\beta} = A_\beta \cdot \left(\frac{I_{q,\beta}}{\sum_q I_{q,\beta}} \right)$$

$\mathbf{X_{q,\beta}}$ is the level of the QNA estimate for quarter q of year β

$\mathbf{I_{q,\beta}}$ is the level of the indicator in quarter q of year β

$\mathbf{A_{\beta}}$ is the level of the ANA estimate for year β

Pro Rata Distribution and Step Problem

Benchmark-to-indicator ratio presentation

$$X_{q,\beta} = I_{q,\beta} \cdot \left(\frac{A_\beta}{\sum_q I_{q,\beta}} \right)$$

$X_{q,\beta}$ is the level of the QNA estimate for quarter q of year β

$I_{q,\beta}$ is the level of the indicator in quarter q of year β

A_β is the level of the ANA estimate for year β

Bq = A/ Σ Iq is called the "BI ratio" or the "rebasing ratio"

Pro Rata Distribution and Step Problem

- Both equations are algebraically equivalent
- Only the presentation differs
- Pro-rata distribution introduces a discontinuity in the growth rate from the last quarter of one year to the first quarter of the next year - "*step problem*".

Pro-rata method and “step problem”

- BI ratio has to be stable from year to year
- If the BI ratios for adjacent years are very different, a trend break will occur from Q4 to Q1 of the following year. This is known as “step problem”.

Avoiding the step problem

- By smoothing out the changes in the BI ratios
  - BI ratios are treated as quarterly time series which is then smoothened.
  - Apply the smoothened BI series to the indicator series to derive benchmarked series.

Example 6.1. Pro Rata Distribution and Basic Extrapolation

	Indicator				Derived QNA Estimates					
	The Indicator (1)	Period-to-Period Rate of Change	Annual Data (2)	Annual BI ratio (3)	Distributed Data (1)	.	(3)	=	(4)	Period-to-Period Rate of Change
q1 1998	98.2				98.2	.	9.950	=	977.1	
q2 1998	100.8	2.6%			100.8	.	9.950	=	1,003.0	2.6%
q3 1998	102.2	1.4%			102.2	.	9.950	=	1,016.9	1.4%
q4 1998	100.8	–1.4%			100.8	.	9.950	=	1,003.0	–1.4%
Sum	**402.0**		**4000.0**	**9.950**					**4,000.0**	
q1 1999	99.0	***–1.8%***			99.0	.	10.280	=	1,017.7	***1.5%***
q2 1999	101.6	2.6%			101.6	.	10.280	=	1,044.5	2.6%
q3 1999	102.7	1.1%			102.7	.	10.280	=	1,055.8	1.1%
q4 1999	101.5	–1.2%			101.5	.	10.280	=	1,043.4	–1.2%
Sum	**404.8**	**0.7%**	**4161.4**	**10.280**					**4,161.4**	**4.0%**
q1 2000	100.5	–1.0%			100.5	.	10.280	=	1,033.2	–1.0%
q2 2000	103.0	2.5%			103.0	.	10.280	=	1,058.9	2.5%
q3 2000	103.5	0.5%			103.5	.	10.280	=	1,064.0	0.5%
q4 2000	101.5	–1.9%			101.5	.	10.280	=	1,043.4	–1.9%
Sum	**408.5**	**0.9%**	**?**	**?**					**4,199.4**	**0.9%**

Pro Rata Distribution

The annual BI ratio for 1998 of 9.950 is calculated by dividing the annual output value (4000) by the annual sum of the indicator (402.0). This ratio is then used to derive the QNA estimates for the individual quarters of 1998. For example, the QNA estimate for q1 1998 is 977.1, that is, 98.2 times 9.950.

The Step Problem

Observe that quarterly movements are unchanged for all quarters except for q1 1999, where a decline of 1.8% has been replaced by an increase of 1.5%. (In this series, the first quarter is always relatively low because of seasonal factors.) This discontinuity is caused by suddenly changing from one BI ratio to another, that is, creating a step problem. The break is highlighted in the charts, with the indicator and adjusted series going in different directions.

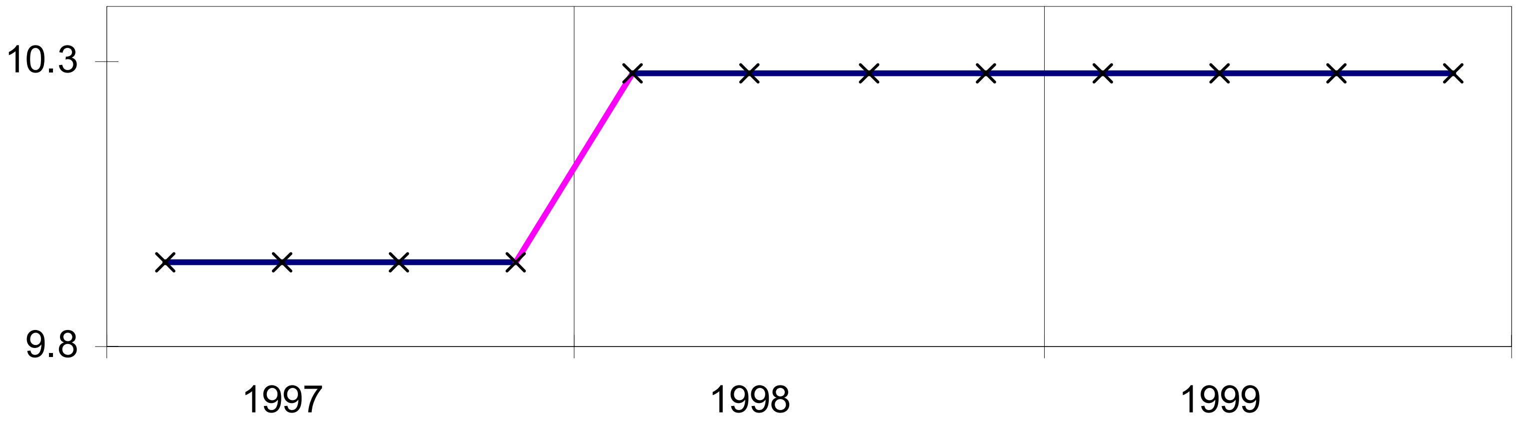
Extrapolation

The 2000 indicator data are linked to the benchmarked data for 1999 by carrying forward the BI ratio for the last quarter of 1999. In this case, where the BI ratio was kept constant through 1999, this is the same as carrying forward the annual BI ratio of 10.280. For instance, the preliminary QNA estimate for the second quarter of 2000 (1058.9) is derived as 103.0 times 10.280. Observe that quarterly movements are unchanged for all quarters.

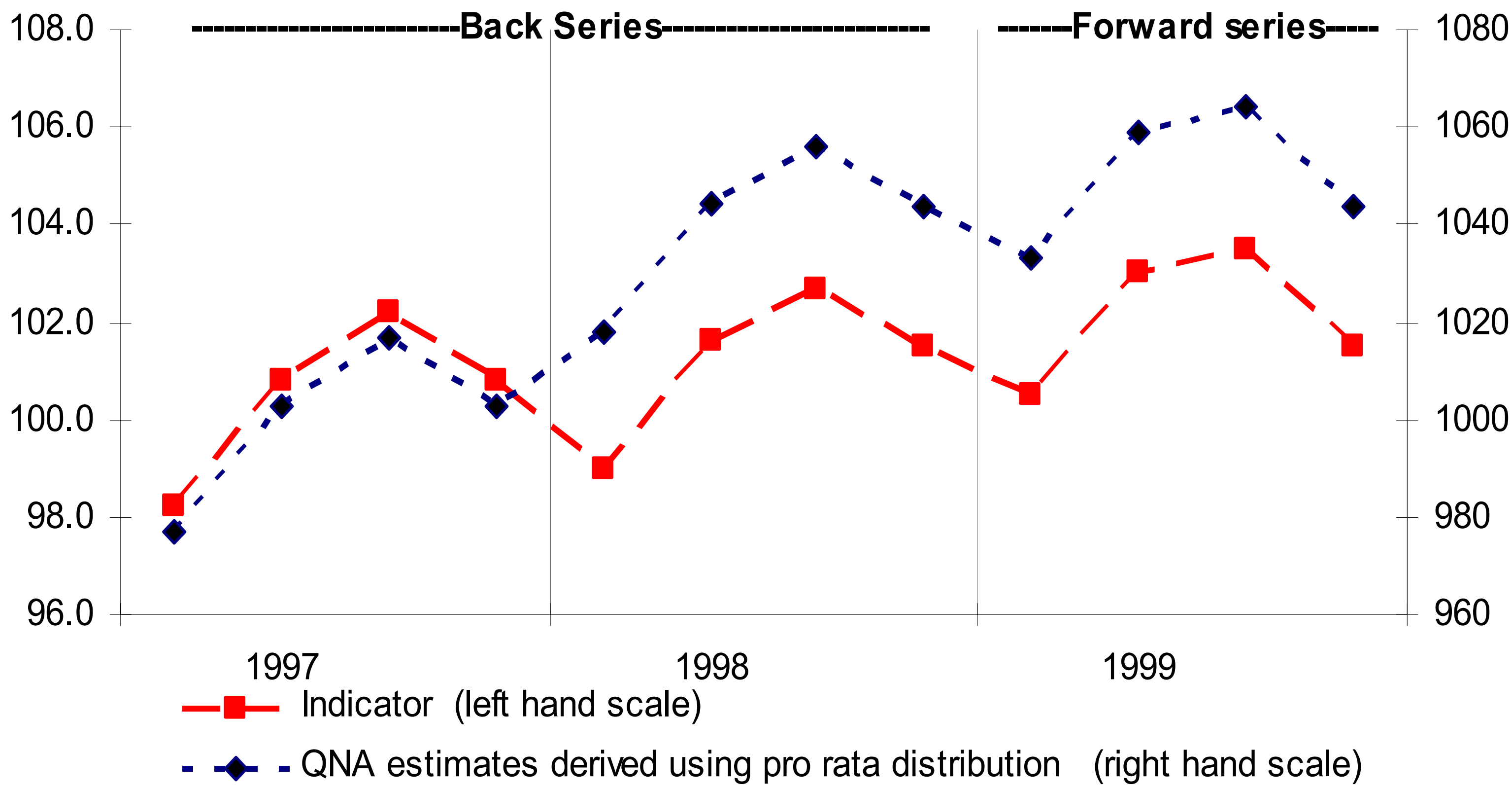
(These results are illustrated in Chart 6.1.)

Pro Rata Distribution: Step Problem

Benchmark-to-indicator ratio



Pro Rata Distribution: Step Problem



Benchmarking Methods that Avoid Steps

- Time series method avoiding steps
  - Various, but same purpose
  - Keeps the movements of the short-term benchmarked series as proportional as possible to those in the original series

Denton method

- Numerical approach
- Least squares minimisation methods
  - The additive Denton (**D_1**) minimises the absolute differences of the absolute adjustments of two neighbouring quarters
  - The proportional Denton (**D_4**) minimises the absolute differences of the relative adjustments of two neighbouring quarters
- **D_4** is preferred over **D_1** as it preserves seasonal fluctuations better.

The Basic Version of the Proportional Denton Method

$$\min_{(X_{1\ldots,}X_{4\beta,\ldots}X_{t})} \sum_{t=2}^{T}\left[\frac{X_t}{I_t} - \frac{X_{t-1}}{I_{t-1}}\right]^2$$

$$t \in \{1..(4\beta),..T\}$$

$$\sum_{t=4y-3}^{4y} X_t = A_y$$

The Basic Version of the Proportional Denton Method

where:

- t is time. For example, t=4y-3 is the first quarter of year y, and $t=4y$ is the fourth quarter of year y
- X_t is the derived QNA estimate for quarter t
- I_t is the level of the indicator for quarter t
- Ay is the ANA estimate for year y
- β is the last year for which an annual benchmark is available, and
- T is the last quarter for which quarterly source data are available

Denton (proportional) method

- The basic version of the proportional Denton benchmarking technique keeps the benchmarked series as proportional to the indicator as possible by minimizing (in a least-squares sense) the difference in relative adjustment to neighbouring quarters subject to the constraints provided by the annual benchmarks
- The proportional Denton technique *implicitly constructs* from the annual observed BI ratios a time series of *quarterly benchmarked QNA estimates-to-indicator* (quarterly BI) ratios that is as smooth as possible
- All quarterly growth rates are adjusted by gradually changing but relatively similar amounts
- Indicators growths are maintained as far as possible
- The sum of adjusted quarterly series adds up to the annual benchmarked values.

Denton (proportional) method

Example 6.2. The Proportional Denton Method

Same data as in Example 6.1.

	Indicator		Annual Data	Annual BI Ratios	Derived QNA Estimates	Estimated Quarterly BI ratios	Period-to-Period Rate of Change
	The Indicator	Period-to-Period Rate of Change					
q1 1998	98.2				969.8	9.876	
q2 1998	100.8	2.6%			998.4	9.905	3.0%
q3 1998	102.2	1.4%			1,018.3	9.964	2.0%
q4 1998	100.8	–1.4%			1,013.4	10.054	–0.5%
Sum	**402.0**		**4000.0**	**9.950**	**4,000.0**		
q1 1999	99.0	*–1.8%*			1,007.2	10.174	–0.6%
q2 1999	101.6	2.6%			1,042.9	10.264	3.5%
q3 1999	102.7	1.1%			1,060.3	10.325	1.7%
q4 1999	101.5	–1.2%			1,051.0	10.355	–0.9%
Sum	**404.8**	**0.7%**	**4161.4**	**10.280**	**4,161.4**		**4.0%**
q1 2000	100.5	–1.0%			1,040.6	10.355	–1.0%
q2 2000	103.0	2.5%			1,066.5	10.355	2.5%
q3 2000	103.5	0.5%			1,071.7	10.355	0.5%
q4 2000	101.5	–1.9%			1,051.0	10.355	–1.9%
Sum	**408.5**	**0.9%**	**?**	**?**	**4,229.8**		**1.6%**

BI Ratios

- For the back series (1998–1999):
  In contrast to the pro rata distribution method in which the estimated quarterly BI ratio jumped abruptly from 9.950 to 10.280, the proportional Denton method produces a smooth series of quarterly BI ratios in which:
  - The quarterly estimates sum to 4000, that is, the weighted average BI ratio for 1998 is 9.950.
  - The quarterly estimates sum to 4161.4, that is, the weighted average for 1999 is equal to 1.0280.
  - The estimated quarterly BI ratio is increasing through 1998 and 1999 to match the increase in the observed annual BI ratio. The increase is smallest at the beginning of 1998 and at the end of 1999.
- For the forward series (2000), the estimates are obtained by carrying forward the quarterly BI ratio (10.355) for the last quarter of 1999 (the last benchmark year).

Rates of Change

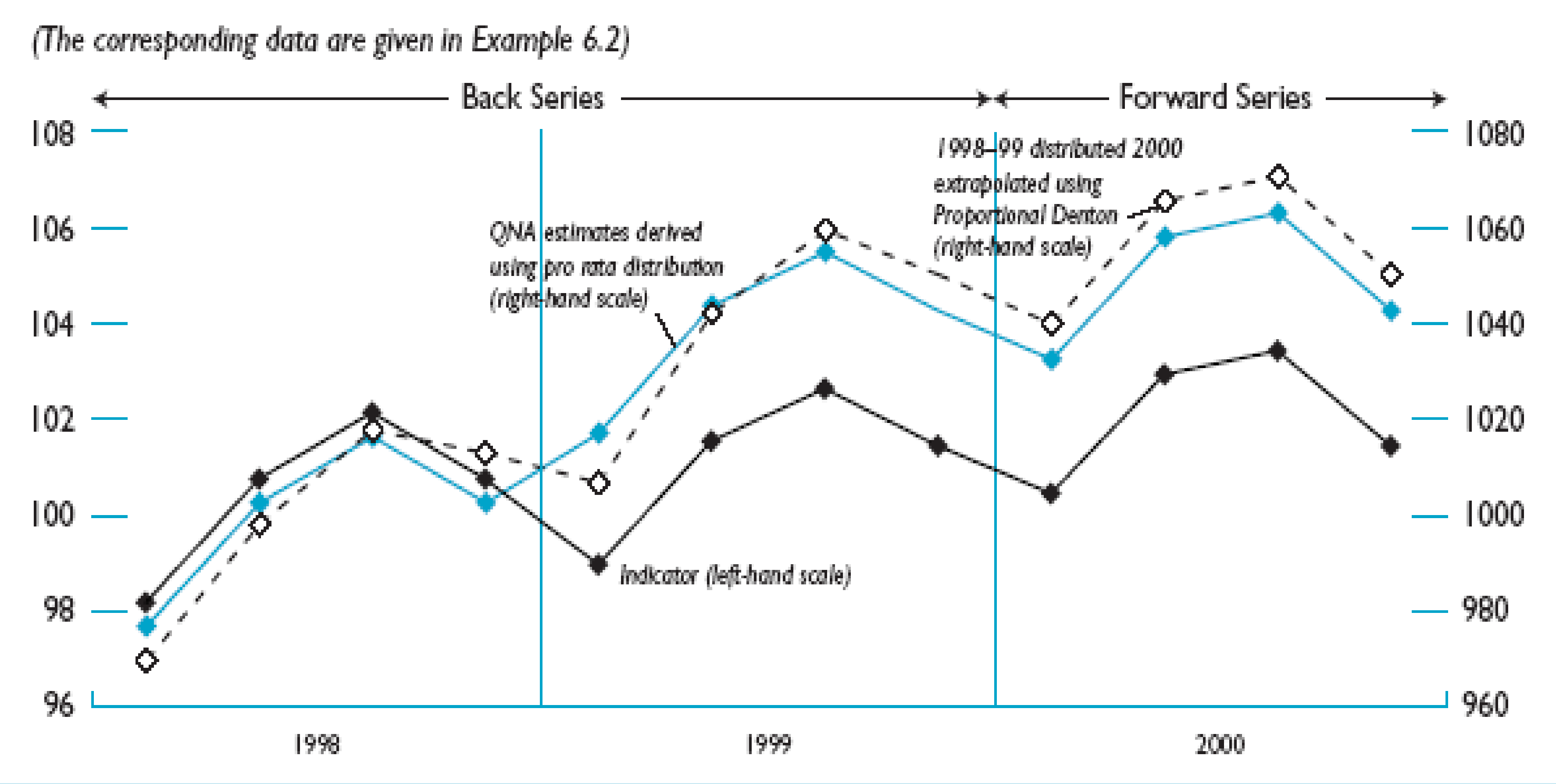
- For the back series, the quarterly percentage changes in 1998 and 1999 are adjusted upwards for all quarters to match the higher rate of change in the annual data.
- For the forward series, the quarterly percentage changes in 1999 are identical to those of the indicator; but note that the rate of change from 1999 to 2000 in the derived QNA series (1.6%) is higher than the annual rate of change in the indicator (0.9%). The next section provides an extension of the method that can be use to ensure that annual rate of change in the derived QNA series equals the annual rate of change in the indicator, if that is desired.

(These results are illustrated in Chart 6.2.)

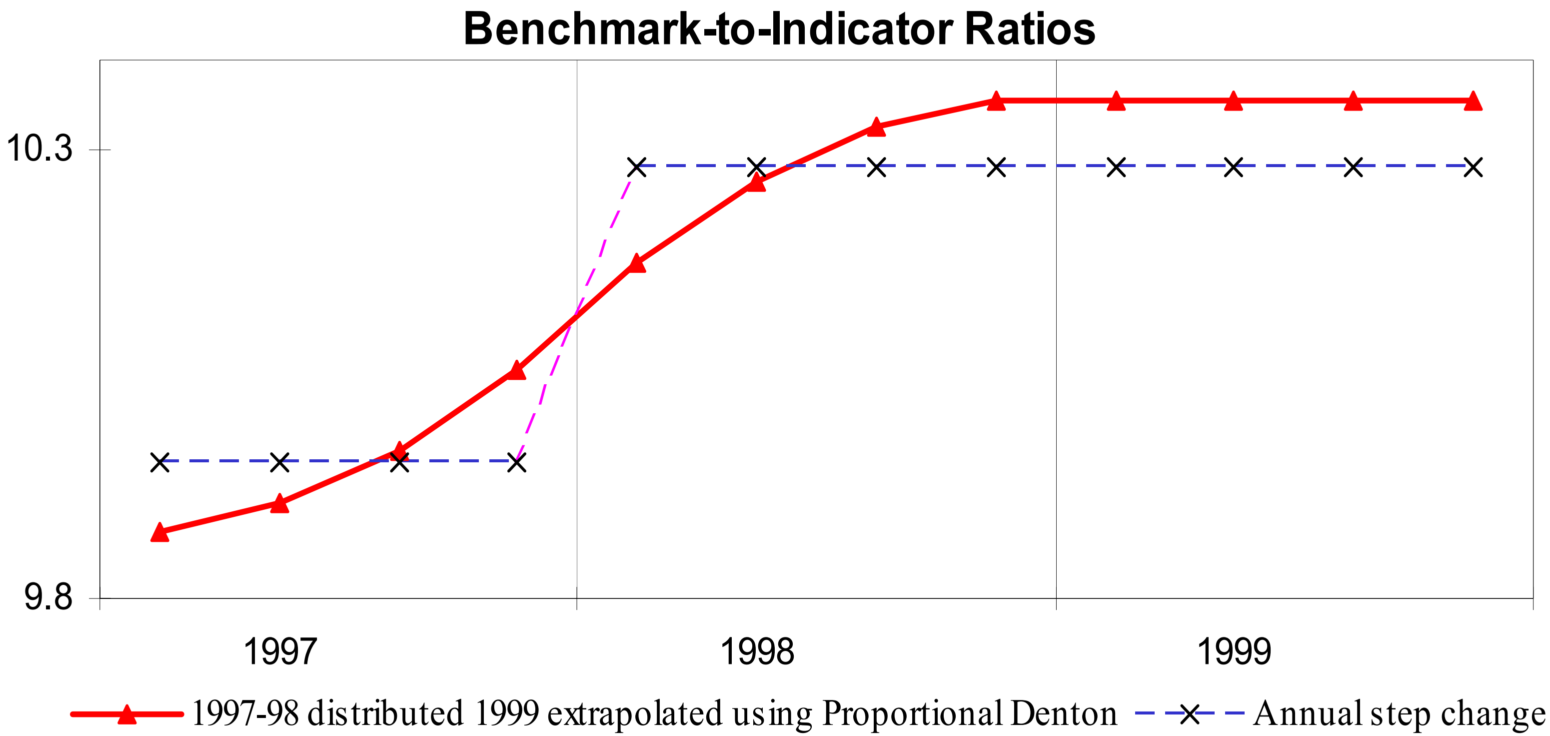
Denton (proportional) method

Chart 6.2. Solution to the Step Problem: The Proportional Denton Method

The Indicator and the Derived Benchmarked QNA Estimates



Proportional Denton Method



Benchmarking and Revisions

- For recent quarters with no ANA data, the proportional Denton method implies moving the fourth quarter of the last benchmarked year by the indicator series

- This is equivalent to using B-I ratio for the fourth quarter of the last benchmark year to scale up or down the quarterly indicator

The Bassie method

The method is as follows:

1. Select a pair of two years for benchmarking.

2. Apply the simple prorating method to the original quarter data of the first year in the pair.

3. Apply the following formula for adjusting the prorated data of the first year and the original data of the second year as follows:

Find the difference between the annual value of the second year and the sum of quarter data:

$$D_2 = A_2 - \sum X_{q,2}$$

Find the new adjusted value of the quarters for year 1 and year 2

$$Z_{q1} = X_{q,1} + 0.25 \times b_q \times D_2$$

$$Z_{q,2} = X_{q,2} + 0.25 \times c_q \times D_2$$

Subscript 1,2 refer to the first and second year.

The Bassie method

The value of b and c are as follows:

To be used for the first year		To be used for the second year	
b1	-0.0981445	c1	0.57373047
b2	-0.1440297	c2	0.90283203
b3	-0.0083008	c3	1.17911122
b4	0.25048828	c4	1.34423822
Sum	0.0		4.0

Thank You