

Multiple Imputations of Missing Data in the Environmental Sustainability Index - Pain or Gain?

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Abstract

The Environmental Sustainability Index (ESI) is a composite index, aggregating data on the national level to measure a country's performance in sustaining a healthy, livable environment. The resulting country ranking thus serves as a barometer for environmental sustainability.

The paper addresses the problem of missing data in the construction of the index. Multiple imputation methods have been shown to avoid problems such as selection bias, underestimation and information loss associated with ad-hoc solutions like case deletion, mean substitution, best guess imputation, or regression methods.

Multiple imputations using Markov Chain Monte Carlo (MCMC) techniques are applied to substitute missing observations with plausible pseudorandom samples from the conditional probability distribution of the missing data given the observed values. The indices calculated from the imputed datasets are combined to result in a final ESI ranking. Multiple imputations using MCMC are shown to yield similar results compared to the 2001 ESI ranking. An estimate of the variance of the index for each country, however, indicates that the presentation of the country ranking alone can be misleading.

¹ The findings, interpretations and conclusions expressed in this paper are entirely those of the author and should not be attributed in any manner to the United Nations, to its affiliated organizations, or the countries they represent.

Keywords: missing values, multiple imputations, MCMC, composite index

1. Introduction

The Environmental Sustainability Index (ESI) is a measure of the overall progress towards environmental sustainability, representing a first step towards a more analytically driven approach to environmental decision-making (2001 ESI report). The question of the feasibility of measuring sustainability is heatedly debated and many issues remain to be solved. This paper focuses on the problem of missing data in the calculation of the index. We adopt the index methodology for the purpose of comparing the results from the imputations with the 2001 ESI but acknowledge that the methodology itself calls for further improvement.

We propose the application of multiple imputations using Markov Chain Monte Carlo simulation (MCMC) as an alternative to the single imputation algorithm used in the ESI 2001. Multiple imputations account for the uncertainty associated with the missing values and provide a unified methodology, which under certain model assumptions, leads to valid statistical inference.

The paper is structured as follows: in section 2 the index methodology is described and in section 3 we derive the MCMC approach. In section 4 the resulting ESI scores based on the imputed datasets are compared to the 2001 ESI. An estimate for the standard errors of the ESI scores is derived in Section 5. The paper concludes with a summary of the findings and some remarks on the interpretability of the ESI.

2. The Environmental Sustainability Index

Environmental sustainability is defined along 5 dimensions: *Environmental systems*, *Reducing environmental stress*, *Reducing human vulnerability*, *Social and institutional capacity*, and *Global stewardship* (2001 ESI report). The full definitions of the dimensions are given in Annex 1. Allocated to these 5 dimensions are 22 indicators, each of which is the equally weighted average of 2 to 6 variables of a total

of 67 underlying variables (65 continuous, 2 categorical). The variables have been carefully selected according to a) extensive expert consultation and analysis, b) literature review, and c) data availability (2001 ESI report). The final index is based on the equally weighted average of the 22 indicators and covers 122 countries in 2001. A high ESI score indicates that a country has achieved a higher level of environmental sustainability than a country with a low score.

Before imputation, the data were transformed to be more comparable. Where possible, variables were adjusted for GDP, population or populated land area (area with at least 5 inh/km²) and then standardized using the sample mean and sample standard deviation of the incomplete data such that large observed values correspond to a high level of environmental sustainability. Highly skewed variables were log-transformed. Thresholds were imposed on 2 variables since countries exceeding these thresholds were not considered more sustainable: an upper threshold of 120 per cent on Daily per capita calorie supply as per cent of total requirements and a lower threshold of 0 on Projected change in population 2000-2050.

After imputation, the observations in the completed dataset were truncated, forcing the bottom 2.5 and top 97.5 percentiles to equal the 2.5 and 97.5 percentile, respectively. The reasons are i) weak confidence in the accuracy of the data in the extreme tails of the distribution and ii) to prevent extreme outliers from becoming benchmarks for the entire population (2001 ESI report).

More generally, let $Y_1, Y_2, \dots, Y_P, p=1, 2, \dots, P$, define the standardized and truncated variables that compose the index, *ESI*, through the indicators $I_1, I_2, \dots, I_J, j=1, 2, \dots, J$. The index is calculated for $N, n=1, 2, \dots, N$, countries. Let S define an indicator matrix with elements $s_{j,p}$, such that $s_{j,p}$ is 1 if Y_p is in the j th indicator and 0 otherwise.

The ESI for the n th country is then defined as:

$$ESI^{(n)} = \frac{1}{J} \sum_{j=1}^J I_j = \frac{1}{J} \sum_{j=1}^J \frac{1}{s_{j..}} \sum_{p=1}^P s_{j,p} Y_p^{(n)} \quad (2.1)$$

$$\text{where } s_{j,\cdot} = \sum_{p=1}^P s_{j,p} . \quad (2.2)$$

This ESI score is then converted to a standard normal percentile to give the index greater intuitive meaning (2001 ESI).

3. Multiple imputations using MCMC simulation

When the proportion of missing values is relatively small, i.e. less than 5 per cent, ad-hoc methods such as case deletion and mean substitution may be an appropriate solution. In multivariate settings, however, where one or more variables might be missing, the proportion of cases with missing values can be substantial. Besides the loss of information, case deletion can lead to biased results if the incomplete cases differ systematically from the complete cases. Mean substitution, although preserving the variable means, leads to underestimation of the variance-covariance structure, whereas regression methods for imputation tend to inflate observed correlations. Multiple imputation (MI) using MCMC simulations reflects the uncertainty associated with the missing observation, providing unbiased estimates for the parameters of interest and their variances.

MI essentially consists of 3 steps:

1. Create M complete datasets by filling in the missing values through imputation.
2. Analyze the M completed datasets.
3. Combine the results from the M analyses to yield final inference on the parameters of interest.

Let Y denote the $(N \times P)$ data matrix for P variables and N countries, which can be partitioned into the observed data, Y_{obs} , and the missing data, Y_{mis} . Multiple imputation using MCMC is based on the assumptions that i) the data rows are iid, ii) the model for the imputations has been correctly specified up to the vector θ of unknown model parameters, and iii) the missing data are missing at random (MAR), i.e. if R is the $(N \times$

P) indicator matrix with elements $r_{n,p}$, where $r_{n,p}$ is 1 if $Y_{n,p}$ is observed and 0 otherwise, and ξ denotes unknown parameters governing the distribution of R , then

$$P(R | Y_{obs}, Y_{mis}, \xi) = P(R | Y_{obs}, \xi) \quad (3.1)$$

Furthermore, the missingness mechanism is said to be ignorable if in addition to the MAR assumption, the parameters ξ of the missingness process and θ of the data model are distinct. According to Schafer (1997), under ignorability, we do not need to consider the distribution of R when making Bayesian inference about θ .

Assuming that the data come from a multivariate normal distribution with mean vector μ and covariance matrix Σ , the data model parameter is $\theta = (\mu, \Sigma)$. The distribution of interest is the joint posterior probability distribution of θ and the missing observations Y_{mis} given the observed data, i.e. $P(Y_{mis}, \theta | Y_{obs})$. In order to sample from this distribution, we construct a Markov Chain whose stationary distribution is $P(\theta, Y_{mis} | Y_{obs})$. The observed quantities are Y_{obs} and R . Inference on θ in a Bayesian framework, assuming ignorability and defining $\pi(\theta)$ as the prior distribution of θ , is thus based on the marginal posterior for θ , or observed-data posterior

$$P(\theta | Y_{obs}) \propto L(\theta | Y_{obs})\pi(\theta) . \quad (3.2)$$

In order to make sampling from the observed-data posterior feasible, it is augmented by plausible values for Y_{mis} , sampled from $P(Y_{mis} | Y_{obs}, \theta)$ for some fixed θ . The simulation algorithm is thus given by:

1. The imputation step (I-step): Given a current estimate $\theta^{(t)}$ of θ (starting values could be the EM estimate of the MLE, for example), simulate the missing values $Y_{mis}^{(t+1)}$ independently for each observation from $P(Y_{mis} | Y_{obs}, \theta^{(t)})$.
2. The Posterior step (P-step): Use the simulated values to generate a new sample $\theta^{(t+1)}$ of the mean and covariance from the joint posterior probability

distribution $P(\theta|Y_{obs}, Y_{mis}^{(t+1)})$, based on the completed data likelihood and a prior distribution for θ .

This creates a Markov Chain $Z(t)=\{(Y_{mis}^{(1)}, \theta^{(1)}), (Y_{mis}^{(2)}, \theta^{(2)}), ..., (Y_{mis}^{(t)}, \theta^{(t)}), ...\}$, which under certain mild regularity conditions² converges to the target distribution $P(Y_{mis}, \theta|Y_{obs})$.

After a sufficiently long burn-in period and establishment of convergence of the chain, an approximately independent sample of the missing values is drawn to complete the dataset and the procedure is repeated M times to yield M imputed datasets. According to studies by Rubin (1987), $M=3$ to 5 imputations are sufficient for datasets with small to moderate proportions of missing values. Each imputed dataset is subject to statistical analysis and final inference is based on the combined results.

4. Results

The data rows in the ESI dataset are not exchangeable due to the fact that the data come from countries with varying degrees of economic development, geographical, and socio-cultural characteristics. A cluster analysis, however, did not yield convincing evidence for the existence of distinct country groups. Although the MCMC approach assumes multivariate normality, Schafer (1997) argues that inference tends to be robust to a violation of this assumption if the proportion of missing data is relatively small. With $\lambda=29\%$, the share of missing data in the ESI dataset is moderate. In the case of the ESI, the missingness mechanism is unknown and the MAR assumption cannot be formally tested. However, due to the high correlations between the variables it was possible to predict reasonably well whether an observation was missing or not.

The relative efficiency RE of an estimate of the parameter of interest based on a finite number M of imputations rather than an infinite number is approximately (Rubin 1987)

² The sequence $Z^{(t)}$, $t \rightarrow \infty$, converges to the target distribution if i) the target distribution $P(Z)$ is a genuine probability distribution and the sequence of marginal conditional distributions must be the actual conditional distributions corresponding to the target distribution, ii) sample space of Z is connected, periodicity and absorbing states are not allowed (Schafer 1997).

$$RE = (1 + \frac{\lambda}{M})^{-1}. \quad (4.1)$$

Thus, with $M=5$ and $\lambda=29\%$, we achieve a sufficiently high efficiency of $RE=0.95$.

Multiple imputations were generated using the new experimental PROC MI procedure of SAS 8.01 (SAS is proprietary software of the SAS Institute Inc., Cary, NC, USA). Multiple Markov chains were generated, i.e. one for each imputation, with a burn-in period of $k=1000$. The number of units (122) in the dataset is not substantially larger than the number of variables (67) and many of the variables are highly correlated such that the sample covariance matrix S is nearly singular. Therefore, ridge priors were chosen for the prior distribution $\pi(\theta)$, i.e. $S^*=Diag(S)$, which allows the mean and variances to be estimated from the data and the correlations are moved slightly to 0. Imputation results were validated using i) different random seeds, ii) different initial parameter estimates, e.g. EM estimates based on the posterior modes and MLE, sample mean vector and covariance matrix, and iii) plots of the mean and autocorrelation for selected variables.

The index for the n th country over the M imputed datasets is calculated as the average over the M individual indices.

$$ESI_{(M)}^{(n)} = \frac{1}{M} \sum_{m=1}^M ESI_m^{(n)} = \frac{1}{M} \sum_{m=1}^M \frac{1}{J} \sum_{j=1}^J \frac{1}{s_{j..}} \sum_{p=1}^P s_{j,p} y_p^{(n)} \quad (4.2)$$

The resulting ESI scores (ESI MI) shown in Table 1 are similar to those of the 2001 ESI in a sense that they reflect that highly developed countries tend to rank higher than those at transitional and low stages of economic development. This corresponds to the theory that economic development and national wealth affect but do not necessarily determine environmental sustainability. Economic development in this regard may be considered a necessary but not sufficient condition. However, besides financial resources achieving environmental sustainability requires a supportive environment: a well-developed infrastructure, a strong knowledge and research base,

public awareness and participation, environmentally oriented legislation and effective enforcement of environmental regulations. Armed conflicts, disputes or a generally unstable political situation are additional factors that can negatively influence a country's potential for environmental sustainability.

No	Country	ESI MI	2001 ESI	2001 ESI rank	ESI MI rank	Rank difference
1	Norway	77.25	78.19	2	1	1
2	Finland	75.75	80.47	1	2	-1
3	Sweden	72.16	77.09	4	3	1
4	Canada	71.84	78.14	3	4	-1
5	Switzerland	68.18	74.61	5	5	0
6	Iceland	67.44	67.32	9	6	3
7	Austria	65.67	65.98	8	7	1
8	Netherlands	64.98	70.73	12	8	4
9	Denmark	64.24	63.2	10	9	1
10	Australia	63.27	67.02	7	10	-3
62	Bhutan	48.41	47.99	75	62	13
63	Macedonia, FYR	47.88	51.84	100	63	37
64	Kazakhstan	47.65	49.84	91	64	27
65	Moldova	47.14	46.47	59	65	-6
66	Korea, Rep.	47.00	41.58	95	66	29
67	South Africa	46.97	37.56	45	67	-22
68	Morocco	46.72	52.01	89	68	21
69	Uzbekistan	46.46	41.63	90	69	21
70	Ecuador	46.32	43.91	44	70	-26
71	Sri Lanka	46.22	51.95	51	71	-20
113	Togo	39.02	31.24	101	113	-12
114	Burkina Faso	38.72	39.45	104	114	-10
115	Philippines	38.04	38.56	112	115	-3
116	Burundi	37.55	30.07	120	116	4
117	El Salvador	36.03	33.46	84	117	-33
118	Rwanda	35.13	35.68	115	118	-3
119	Madagascar	34.85	43.7	113	119	-6
120	Ethiopia	33.07	35.37	119	120	-1
121	Nigeria	32.56	31.81	117	121	-4
122	Haiti	25.88	24.71	122	122	0

Table 1: The ESI based on multiply imputed data (ESI MI) vs. the 2001 ESI (2001 ESI) for the top, middle and bottom 10 countries.

The variation in the ranking for the 2001 ESI and the ESI MI is larger for those countries with low and medium ranks compared to those with top ranks. This is due to

the fact that, generally, the corresponding countries have a higher proportion of missing values than the top ranking countries, hence higher uncertainty about the true ESI value. For example, if the countries are divided into 3 groups according to GDP/cap: developed countries, economies in transition and developing countries, then the average proportion of missing data for each group is 0.14, 0.21, and 0.34, respectively. Furthermore, the estimated individual index values are not very dispersed, especially for the medium ranks, small changes in the ESI can thus lead to substantial changes in the ranks.

Figure1 shows the relationship between the ESI MI scores data and GDP/capita. A linear regression yields $R^2=0.62$ and the correlation of $\rho=0.79$ is significant at the $\alpha=0.05$ level. However, a high GDP/cap does not imply a high ESI score as the variation in the plot illustrates.

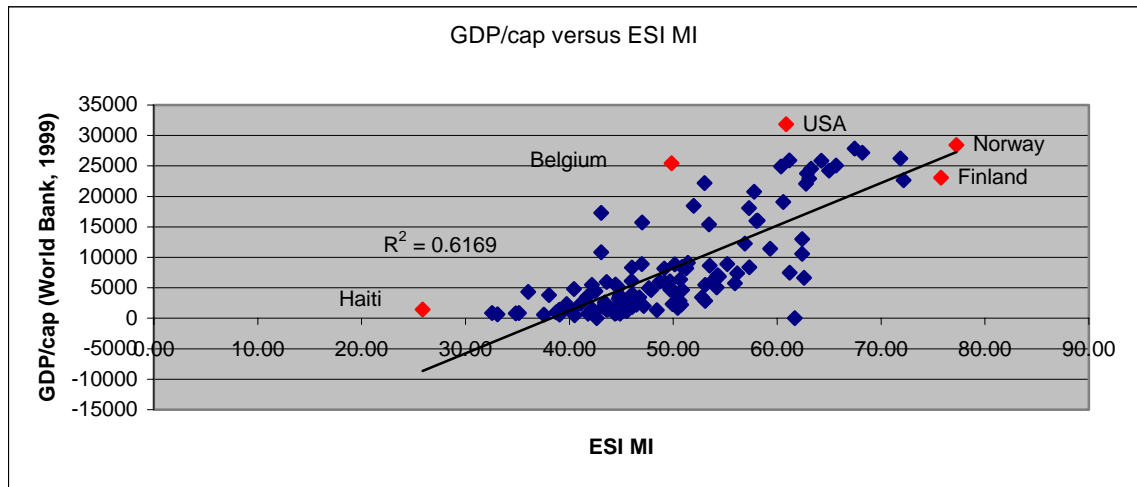


Figure 1: Relationship between the ESI MI and World Bank GDP/cap figures for 1999 (Source: United Nations Common Database).

The relationship between the ESI and the World Economic Forum's Competitiveness indices as shown in Figure 2 and 3 is less strong than between the ESI and GDP/cap but it is clearly visible that competitiveness positively correlates with environmental sustainability. The correlations are 0.70 and 0.69 for current competitiveness and growth competitiveness, respectively.

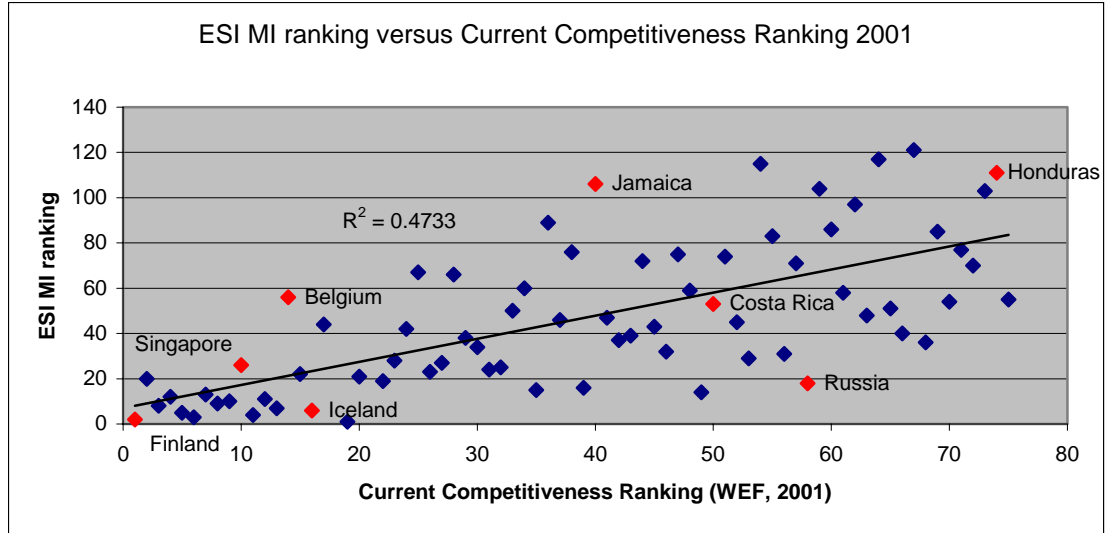


Figure 2: Relationship between ESI MI ranking and Current Competitiveness Ranking 2001 (Source: World Economic Forum, Current Competitiveness Ranking 2001).

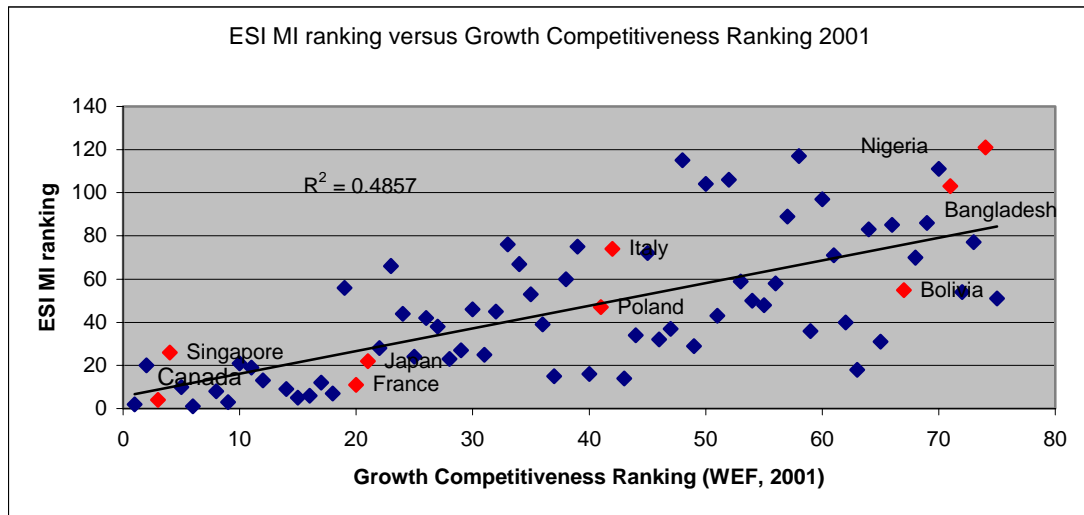


Figure 3: Relationship between ESI MI ranking and Growth Competitiveness 2001 (Source: World Economic Forum, Growth Competitiveness Ranking 2001).

5. Estimation of the variance of the ESI

The ranking of the countries according to their environmental sustainability index should be analyzed in conjunction with an estimate of the accuracy of the individual index scores. For this purpose, an estimate of the variance, $\hat{s}_{(ESI_{(M)}^{(n)})}$, is derived, taking into account the data model assumptions and the uncertainty associated with the imputed values.

Using (4.2) we have

$$\hat{S}(ESI_{(M)}^{(n)}) = Var \left[\frac{1}{M} \sum_{m=1}^M \frac{1}{J} \sum_{j=1}^J \frac{1}{s_{j\cdot}} \sum_{p=1}^P s_{j,p} y_p^{(n)} \right] = \frac{1}{(MJ)^2} \sum_{m=1}^M \sum_{j=1}^J \frac{1}{s_{j\cdot}^2} \sum_{p=1}^P s_{j,p} Var(y_p^{(n)}) . \quad (5.1)$$

The variance of Y_p depends on whether this variable was observed for the n th country. If Y_p was observed, then it has only the data-model variance associated with it, if it was missing and has been imputed M times, it is also subject to variation due to its missingness. According to Rubin (1987), the combined variance can be partitioned into 2 components: the within-imputation variance, which is the average of the M complete-data variances for Y_p , and the between-imputation variance, which is estimated by the sample variance of Y_p over the M imputations adjusted for the fact that M is small. Thus, using the missingness indicator matrix R , defined in section 3, (5.1) becomes

$$\hat{S}(ESI_{(M)}^{(n)}) = \frac{1}{(MJ)^2} \sum_{m=1}^M \sum_{j=1}^J \frac{1}{s_{j\cdot}^2} \sum_{p=1}^P s_{j,p} \left(\frac{1}{M} \sum_{m=1}^M \hat{\sigma}_{p,m}^2 + (1 - r_{n,p}) \left(1 + \frac{1}{M} \right) \left(\frac{1}{M-1} \sum_{m=1}^M (y_p^{(n)} - \bar{y}_p^{(n)})^2 \right) \right) \quad (5.2)$$

Based on the assumption of multivariate normality and the standardization of the variables, a confidence interval for $\alpha=0.05$ for the n th country is then given by

$$\left[ESI_{(M)}^{(n)} - \sqrt{\hat{S}(ESI_{(M)}^{(n)})} u_{(1-\alpha/2)}; ESI_{(M)}^{(n)} + \sqrt{\hat{S}(ESI_{(M)}^{(n)})} u_{(1-\alpha/2)} \right]. \quad (5.3)$$

The results in Table 2 show that the confidence intervals are relatively wide, the average range between the upper and lower confidence boundary is 29. Countries with small amounts of missing data have smaller confidence intervals than countries with many missing values and those whose ranks varied considerably between the 2001 ESI and the ESI MI tend to have a large number of missing variables, more than 30 out of 67, and particularly wide confidence intervals. Hence, one should be careful to make conclusions on the degree to which countries have achieved environmental sustainability based on the country ranking alone.

Rank	Country	Lower 95% CI limit	ESI MI	Upper 95% CI limit
1	Norway	66.78	77.25	85.55
2	Finland	65.89	75.75	83.83
3	Sweden	60.59	72.16	81.77
4	Canada	61.71	71.84	80.47
5	Switzerland	56.80	68.18	78.06
6	Iceland	50.24	67.44	81.55
7	Austria	53.34	65.67	76.53
8	Netherlands	53.79	64.98	75.00
9	Denmark	50.38	64.24	76.44
10	Australia	49.38	63.27	75.61
62	Bhutan	31.88	48.41	65.21
63	Macedonia, FYR	33.31	47.88	62.74
64	Kazakhstan	32.67	47.65	62.98
65	Moldova	31.13	47.14	63.63
66	Korea, Rep.	34.61	47.00	59.69
67	South Africa	33.40	46.97	60.91
68	Morocco	31.22	46.72	62.75
69	Uzbekistan	31.04	46.46	62.43
70	Ecuador	28.70	46.32	64.71
71	Sri Lanka	29.95	46.22	63.15
113	Togo	22.97	39.02	57.23
114	Burkina Faso	25.24	38.72	53.74
115	Philippines	25.57	38.04	51.91
116	Burundi	24.12	37.55	52.70
117	El Salvador	20.80	36.03	53.90
118	Rwanda	19.95	35.13	53.18
119	Madagascar	21.75	34.85	50.07
120	Ethiopia	20.12	33.07	48.47
121	Nigeria	20.73	32.56	46.48
122	Haiti	13.83	25.88	41.85

Table2: 95% confidence interval for the ESI MI for the top, medium and bottom 10 countries.

6. Conclusions

The Environmental Sustainability Index represents an attempt to measure environmental sustainability in a quantitative way. The resulting country ranking can serve as a preliminary indicator of whether a country is moving in the ‘right direction’ as laid out in Agenda 21 of the World Summit on Environment and Development in 1992 in Rio de Janeiro and is thus a useful tool for decision-making. It may also help

to raise the interest of policy-makers and the public to engage more actively in the process of identifying what sustainable development means and how sustainable environments can be created and preserved. Sustainability is not a static concept but rather a process and so is the ESI. It is a relative measure, comparing countries at their respective stages of progress, not defining boundaries of sustainability.

However, one needs to be aware of the limitations of the index. There are conceptual as well as technical issues that remain to be solved. A commonly accepted and measurable definition of what environmental sustainability means has not yet evolved and thus the variable selection, weighting and aggregation methodologies need further discussion. A single measure of environmental sustainability represents an extremely condensed reflection of a complex concept. The index group is therefore working on the development of environmental performance indices for the natural spheres atmosphere, water, land/soils and habitats.

Data availability and quality is still a limiting factor for the precision of the ESI. MCMC methods are useful for the plausible substitution of missing data. The index data pose several problems to its application though. The units are not iid and the MAR assumption cannot be validated. Multicollinearity requires adjustment of the covariance matrix to avoid singularity. The resulting ESI scores are difficult to validate objectively. They can be compared to other relevant indices and to independent expert opinions. The ESI MI supports the findings of the 2001 ESI and the concept of economic development as a pre-requisite for environmental sustainability.

The estimated variance of the individual country scores show that the ESI is not precise enough to base conclusions on the country rankings alone. This conclusion is supported by the complexity of environmental sustainability as well as the current lack of sufficient information and accurate data to measure it.

Annex 1

The 5 dimensions of the ESI are defined as follows:

Environmental systems: A country is environmentally sustainable to the extent that its vital environmental systems are maintained at healthy levels, and to the extent to which levels are improving rather than deteriorating.

Reducing environmental stresses: A country is environmentally sustainable if the levels of anthropogenic stress are low enough to engender no demonstrable harm to its environmental systems.

Reducing human vulnerability: A country is environmentally sustainable to the extent that people and social systems are not vulnerable (in the way of basic needs such as health and nutrition) to environmental disturbances; becoming less vulnerable is a sign that a society is on a track to greater sustainability.

Social and institutional capacity: A country is environmentally sustainable to the extent that it has in place institutions and underlying social patterns of skills, attitudes and networks that foster effective responses to environmental challenges.

Global stewardship: A country is environmentally sustainable if it cooperates with other countries to manage common environmental problems, and if it reduces negative extra-territorial environmental impacts on other countries to levels that cause no serious harm.

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