Premature Mortality and Poverty Measurement

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Abstract

There is a glaring paradox in all commonly used measures of poverty. The death of a poor person reduces poverty according to these measures. This surely violates our basic intuitions of how poverty measures should behave. It cannot be right in concept that differentially higher mortality among the poor serves to reduce poverty. This paper begins the task of developing poverty measures that are not perversely mortality sensitive. A family of measures is proposed that is an intuitive modification of standard poverty measures to take into account the fact that the rich live longer than the poor.

Key words: *Premature Mortality, Life Time Income Profile, Poverty Measure, Characterization, Steady State Population*

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1 Introduction

More than a quarter of a century ago, **Amartya Sen (1976)** pointed out a glaring paradox in the most commonly used measure of poverty - the head-count ratio. He observed that taking income away from the poor did not change this measure, since it did not change the number of people in poverty. This, he argued, violated our basic intuitions about poverty. This simple but powerful observation led to the development of "distribution sensitive" poverty measures, including the Sen measure (**Sen, 1976**) and the FGT measure (**Foster, Greer and Thorbecke, 1984**), which have become the workhorse poverty measures in applied and policy work.

But there is perhaps an even more glaring paradox in the head-count ratio and it is this. If a poor person dies, poverty decreases. This also holds true for the distribution sensitive measures of poverty such as the commonly used FGT family of measures. Reduction of poverty through deaths of the poor must surely violate our basic intuitions on poverty. It cannot be right in concept that differentially higher mortality rates among the poor serve to reduce poverty. This conceptual challenge is only strengthened by the fact that higher mortality rates and lower life expectancies among the poor are an established empirical regularity the world over. Similar issues are discussed in the area of health economics also. An untimely demise of the poor, who are usually not in a very good state of health, has the perverse implication that the average health standards of the society improves.

Of course, the paradoxes of population variation and welfare measurement have exercised philosophers and economists over many years. **Parfits (1984)** "Repugnant Conclusion" launched the modern debate. As formulated by **Arrehnius (2000)**, this is a critique of Total Utilitarianism since: "For any perfectly equal population with very high positive welfare, there is a population with a very low positive welfare which is better." What is repugnant is that one society can be pronounced to be better than another even though every person in the former is worse off than every person in the latter, simply because population in the first is so much higher than in the second. Issues of length of life and standard of living during life are also discussed in the economics literature, for example in **Blackorby, Bossert and Donaldson (1999)**. Focusing on the average utility as a way of getting around the repugnant conclusion has its own problems because, as **Cowen (1989)** rightly observes, "average utilitarianism cannot escape recommending the death of all those below the social mean." In the case of standard poverty measures, this paradox manifest itself through the conclusion that killing off people below the poverty line will reduce poverty.

This paper draws its inspiration from the large literature on population and welfare measurement, but its focus is on the measurement of poverty. Its objective is to launch a discussion that will, hopefully, lead to the development of poverty measures that are not vulnerable to the mortality paradox. We begin in Section 2 with a simple and intuitive modification of the FGT class of poverty measures, to illustrate the nature of our arguments. Section 3 starts the formal and rigorous analysis with a statement of notation and definitions, and some preliminaries. Section 4 axiomatizes lifetime poverty measures, and Section 5 introduces premature mortality. It will be seen that the simple and intutive discussion of Section 2 falls out as a special case of the more general results developed in Section 5. Finally, Section 6 offers some concluding remarks.

2 The Relevant Set of Individuals and a Simple Modification of FGT

The standard approaches to poverty measurement look at a snapshot of alive individuals. Hence the paradox that when a poor individual disappears, measured poverty goes down. The obvious answer to this paradox is to not let individuals disappear because of death, to keep them nevertheless in the universe of individuals whose poverty is being measured. We call the set of individuals who enter into the measurement poverty, whether they are alive or dead, the relevant set of individuals.

It should be clear that the relevant set of individuals is a deeply normative concept. Who should we consider? All those individuals who have ever lived? Should deaths due to poverty in the middle ages burden poverty measurement today? We believe that a good starting point is to specify a normative lifetime L, close to the top of the range observed today in rich countries. The exact value is arbitrary, but a range around 80 years seems reasonable, especially if our focus is on poor countries. Then one specification of the relevant set today is all those individuals who would have been alive today had they lived to the age of 80. In other words, the relevant set is all individuals born L years ago or later.

Keeping with the tradition of measuring poverty on a snapshot income distribution, we would next need to specify the income today of the relevant set of individuals specified above. For those currently alive, of course, it is their current income. The interesting conceptual question is, what income do we ascribe to those who died prematurely? In principle we should project what they would have had if they had been alive and, with this information, calculate any of the FGT family of poverty indices, for example. This is not easy to do empirically, but conceptual clarity can be gained by considering a situation in which the n income levels, $Y_1, Y_2, ..., Y_n$, remain constant over time, and there is no mobility across income levels. Further, suppose the population distribution is also in a steady state in the following sense. Each individual at income level Y_i lives for l_i periods, after which time he or she is replaced by exactly one individual. The observed snapshot distribution of income thus has one individual at each of the income levels Y_i, and the FGT index can be calculated for this in the usual way.

However, now consider the relevant set of individuals. This includes all those who were born L years ago or less. Define $I(x)$ as a function that finds the closest higher integer to x. At income level Y_i there is currently 1 individual alive, but also $I(L/l_i) - 1$ individuals who would have been alive had they lived the full L years of the normative lifespan. Hence there are $I(L/l_i)$ individuals in the relevant set at income level Y_i, and $N = \sum_i I(L/l_i)$ individuals in the relevant set in all.

If the standard axioms of poverty measurement are now invoked, any of the corresponding poverty measures can be defined on the relevant set. The FGT family of indices thus becomes:

$$P^l = \frac{1}{N}\sum_i I(L/l_i)(\frac{z - Y_i}{z})^\alpha,$$

where z is the poverty line and the superscript l indicates adjustment for differential lengths of life across income levels. It can be seen from the above that if l_i is constant across Y_i then we recover the standard FGT index. However, if income and length of life are positively correlated, the P^l measure will be higher than the traditional poverty measure P and if there is negative correlation the P^l measure will be lower than P. In any event, measured poverty will be affected by the income lifetime relation, over and above the distribution of income. The measure given above is easily implementable given the wealth of information on the relationship between income and expected length of life.

3 Formal analysis: Notation, Definition and Preliminaries

Let us start with some notation. We will denote the set of real number by R and the non-negative reals by R_+. The set of integers is given by Z and the set of positive (negative) integers by Z_+ (Z_-). Let $H = \bigcup_{n \in Z_+} Z_-^n$. We will be considering discrete periods of time in this paper. Denote $\bigcup_{n \in Z_+} R_+^n$ by Ω, the set of all possible income distributions, R_+^n being the n-dimensional cartesian product of R_+.

Let there be n individuals in the population under consideration - the relevant set. Each person i is completely characterized by her birth date $t_i \in Z_-$, actual length of life $l_i \in Z_+$ and the life time income profile $Y_i = (y_{i,1}, ..., y_{i,l_i}) \in R_+^{l_i}$ for $i = 1, .., n$, where a typical element $y_{i,l}$ is person i's income in period l of her life. The set $A = \{Y_i, i = 1, ..., n\}$ is the income profile for the relevant population. Denote the vector of birth dates by $T = \{t_1, ..., t_n\}$ where each $t_i \in Z_-$. Thus $T \in H$. To discuss measurement of poverty we need to talk about a measure of subsistence requirement for the population under consideration, or the 'traditional poverty line'. Here, we need to have a subsistence requirement for each period of time. Hence, we define the vector $S = \{..., z_{-1}, z_0, z_1, ...\} \in \Omega$, where the current period is period 0 and $z_t \in R_+$ represents the poverty line for period t.

Let us now define our life time measure of poverty in the most general form as a function

$$P = P(A, T, S, n) : \Omega \times H \times \Omega \times z_+ \rightarrow R_+. \tag{1}$$

So, we assume that the measure of poverty is a positive real value and it depends on the life time income profile for the population, the birth dates of each of the members of the population, the subsistence requirement vector and the size of the population. For person i, the k^{th} period of life is considered to be spent in poverty if $y_{i,k} < z_{t_i+k}$, that is, her income for age k fell below the subsistence requirement for the period when she was aged k. Note that, now we can not talk about a 'person' being poor; rather, all we can now say is that such and such periods of her life has been spent in poverty. The censored income profile associated with Y_i is denoted by Y_i*, whose typical element is $y_{i,k}* = min\{y_{i,k}, z_{t_i+k}\}$.

The poverty index is supposed to satisfy certain desirable properties. We describe them as follows.

Continuity (C): For all $n \in Z_+$, $S \in \Omega$ and $T \in H$, $P(A, T, S, n)$ is a continuous function of all elements $y_{i,k}$, $k = 1, ..., l_i$ and $i = 1, ..., n$, in A.

Focus (F): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$ and $B = (X_1, X_2, ..., X_n)$ with elements $y_{i,k} = x_{i,k}$ whenever $y_{i,k} < z_{t_i+k}$ and $x^t_{i,k} < z_{t_i+k}$, we have $P(A, T, S, n) = P(B, T, S, n)$.

Monotonicity (M): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$ and $B = (X_1, X_2, ..., X_n)$ with elements $y_{i,k} = x_{i,k} < z_{t_i+k}$

for all $i \neq j$, $y_{j,k} = x_{j,k} < z_{t_i+k}$ for all $k \neq l$ and $y_{j,l} < x_{j,l} < z_{t_i+k}$, we have $P(A,T,S,n) > P(B,T,S,n)$.

Symmetry (S): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, if $B = (Y_{\pi(1)}, Y_{\pi(2)}, ..., Y_{\pi(n)})$ and $T' = (t_{\pi(1)}, t_{\pi(2)}, ..., t_{\pi(n)})$, where $(\pi(1), \pi(2), ..., \pi(n))$ is any permutation of $(1, 2, ..., n)$, then $P(A,T,S,n) = P(B,T',S,n)$.

Scale Invariance (SI): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, $P(A,T,S,n) = P(A',T,S',n)$ where A' is a set of income profiles $(X_1, ..., X_n)$ with birth dates T and $S' = \{..., z'_0, ...\}$ is a subsistence requirement vector with the property that for all i and for all t, $\frac{y_{i,t}}{z_{t_i+t}} = \frac{x_{i,t}}{z'_{t_i+t}}$.

Translation Invariance (TI): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, $P(A,T,S,n) = P(A',T,S',n)$ where $A' \in \Omega$ is a set of income profiles $(X_1, ..., X_n)$ with birth dates T and $S' = \{..., z'_0, ...\} \in \Omega$ is a subsistence requirement vector with the property that for all i and for all t, $y_{i,t} - z_{t_i+t} = x_{i,t} - z'_{t_i+t}$.

Population Principle (P): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, if $B = (Y_1, ..., Y_1, Y_2, ..., Y_2, ..., Y_n, ..., Y_n)$ is a m-fold replication of A and T' is the corresponding m-fold replication of T, then $P(A,T,S,n) = P(B,T',S,mn)$.

Interpersonal Transfers Principle (TR): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, if another population income profile B is given by $(Y_1, ..., Y_{i-1}, X_i, Y_{i+1}, ..., Y_{j-1}, X_j, Y_{j+1}, ..., Y_n)$ such that $y_{i,k} = x_{i,k}$ for $k \neq l$ and $y_{j,k} = x_{j,k}$ for $k \neq t = l + (t_i - t_j)$, and $0 < y_{i,l} < x_{i,l} = y_{i,l} + \delta < y_{j,t} - \delta = x_{j,t} < y_{j,t} < z_{t_i+l}$ then $P(A,T,S,n) > P(B,T,S,n)$.

Subgroup Decomposability (D): There exists $Q : R_+ \times R_+ \rightarrow R_+$ such that, for all $n \in Z_+$, $S \in \Omega$ and $T \in H$, if we partition any population income profile $A = (Y_1, Y_2, ..., Y_n)$ into two such matrices $A^1 = (Y_1, Y_2, ..., Y_{n_1})$ and $A^2 = (Y_{n_1+1}, Y_2, ..., Y_n)$, where $1 \leq n_1 \leq n$ and the birth date vector T in to $T^1 =$

$(t_1, ..., t_{n_1})$ and $T^2 = (t_{n_1+1}, ..., t_n)$ in a similar fashion, then

$$P(A, T, S, n) = Q(P(A^1, T^1, S, n_1), P(A^2, T^2, S, (n - n_1))).$$

(C) ensures that minor observational errors in incomes will generate minor changes in the poverty index. (F) says that the poverty index is independent of the incomes in excess of the subsistence requirement. So, person i's income for period l will not affect the poverty measure if she was nonpoor in period l. According to (M), a reduction in the income of a person in any period when she was poor must increase poverty. (S) means that any characteristic other than the life time income profile and birth date, e.g. the names of the individuals, is irrelevant to the measurement of poverty. (SI) says that the poverty index is independent of the unit of measurement for income and subsistence requirement in any period. (TI) says that, for any period, an equal increment in income for all persons and the subsistence requirement do not affect the poverty measure. (P) is the usual replication invariance principle that implies that if we make a m-fold copy of the population, all other things unchanged, then the poverty index is unaffected. (TR) demands that a transfer of income in any period, to person i who is poor in that period from another person j who is also poor in the same period but richer than i, without changing their relative position for that period, will reduce poverty. (D) is similar to the subgroup consistency axiom of **Foster and Shorrocks (1991)** which requires overall poverty for a population partitioned into subgroups to increase if poverty in one or more subgroups increases and stay constant in others. The function Q may be regarded as an aggregate deprivation function where deprivation may be measured in terms of relative or absolute shortfall of income in each period from the subsistence requirement. We can also view this in terms of the censored income profiles. (For a detailed discussion on similar properties related to the usual static, or one period, poverty measurement see **Zheng, 1997**).

So far, we have not discussed any structural assumptions regarding the intertemporal properties of our life time poverty measure. How the different periods of a person's life time are compared is the issue of the next two axioms.

Intertemporal Symmetry (ITS): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$ and for any $l, k \in Z$ such that $t_i < l < k < t_i + l_i$ for all $i = 1, ..., n$, if $A' = (Y_1', Y_2', ..., Y_n')$ with $Y_i' = (y_{i,1}, ..., y_{i,l-1}, y_{i,k}, y_{i,l+1}, ..., y_{i,k-1}, y_{i,l}, y_{i,k+1}$ and $S' = (..., z_{l-1}, z_k, z_{l+1}, ..., z_{k-1}, z_l, z_{k+1}, ...)$, then $P(A, T, S, n) = P(A', T, S', n)$.

Intertemporal Consistency (ITC): For all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, for any $l \in Z$ define

$$A^{\leq} = \{(y_{i,1}, ..., y_{i,l}) \; for \; i \; such \; that \; t_i \leq l \leq t_i + l_i\} \bigcup \{(y_{i,1}, ..., y_{i,l_i}) \; for \; i \; such \; that \; t_i + l_i \leq l\}$$

and

$$A^{>} = \{(y_{i,l+1}, ..., y_{i,l_i}) \; for \; i \; such \; that \; t_i \leq l \leq t_i + l_i\} \bigcup \{(y_{i,1}, ..., y_{i,l_i}) \; for \; i \; such \; that \; l < t_i\}.$$

Also, let $T^{\leq} = \{t_i \in T | t_i \leq l\}$ and $T^{>} = \{t_i \in T | t_i + l_i > l\}$, the cardinality of $T^{\leq}$ be $n^{\leq}$ and that of $T^{>}$ be $n^{>}$. Then there exists $W : R_+ \times R_+ \rightarrow R_+$ such that, $P(A, T, S, n) = W(P(A^{\leq}, T^{\leq}, S, n^{\leq}), P(A^{>}, T^{>}, S, n^{>}))$.

(ITS) is the symmetry requirement across time which ensures that any time period of a person's life time has the same significance with respect to life time poverty calculations. (ITC) is the time line analogue of (D) which requires that for any set of persons, if one partitions the time line into disjoint intervals and does the relevant poverty calculations with each of the individual's so truncated income profiles separately, one can recover the overall life time poverty from these separately computed figures in a consistent manner.

4 Life Time Poverty Measures

In what follow, we will be tackling the issue of how to facilitate the interpersonal aggregation of poverty values first. That is, given the individual poverty status of each person based on their life time income profile, how do we arrive at the aggregate poverty status of the whole population. Also, given the life time income profiles of the members of the population, one has to find a representative value for the feeling of poverty of this person across her life span. That is, one has to postulate a representative poverty level of any person depending on her profile and the subsistence requirement. This is the issue we tackle next. This particular sequencing is for technical ease only and this do not result in any significant loss of generality as we can safely assume that poverty level of any person i in time period k will not in any way interact with the poverty level of another person j $(\neq i)$ in period l $(\neq k)$ in determining the aggregate measure of poverty. so we can safely discuss the aggregation possiblities in this two dimensions (persons and time periods) as two independent issues.

4.1 Interpersonal Aggregation

We will now state the first and basic result of this paper regarding some benchmark features of the poverty measure given by (1) when it is required to satisfy some of the above mentioned properties.

Theorem 1: The poverty measure in (1) satisfies C, F, M, S, P, TR and D if and only if, for all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n) \in \Omega$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n}\phi(f_1(y*_{i,1}, z_{t_i+1}), ..., f_{l_i}(y*_{i,l_i}, z_{t_i+l_i})) \tag{2}$$

where $\phi : \Omega \rightarrow R_+$ is continuous, increasing and convex in each of its arguments. The functions $f_l R_+ \times R_+ \rightarrow R_+$, $l = 1, ..., l_i$, are continuous in the arguments and given the second argument, is decreasing and strictly convex in the first.

Proof : The first part of the proof of this result is similar to that of Proposition 1 and 3 in **Tsui (2002)**, hence, we only present an outline of it. First of all, note that the sufficiency part of this result is very easy to verify. For the necessity part, we proceed as follows.

By property F, we can redefine the poverty measure $P(.)$, in (1), on the censored income profiles Y_i*'s. Property D and S together implies that the aggregate measure $P(.)$ can be written as an aggregate of individual poverty levels $\sum_{i=1}^{n} \phi^1(Y_i*, t_i, S, n)$, where the identical functional form $\phi^1(.)$ is due to (S). We now invoke (P) to arrive at the average form given by

$$\frac{1}{n}\sum_{i=1}^{n} \phi^2(Y_i*, t_i, S)$$

where $\phi^2 : \Omega \times Z_- \times \Omega \to R_+$ is continuous, decreasing and strictly convex in each of the arguments $y_{i,k}*$, $k = 1, ..., l_i$ of Y_i*. The continuity and decreasingness of $\phi^2(.)$ follows from (C) and (M). Finally, convexity of $\phi^2(.)$ in each argument follows from (TR).

Now, let us look at the function ϕ^2 more closely. The arguments t_i are relevant for the poverty calculations only so far as they indicate which element of S to link with any argument of Y_i*. Thus, one can suitably redefine the ϕ^2 function to

$$\phi^3(Y_i*; z_{t_i+1}, ..., z_{t_i+l_i}) : \Omega \times \Omega \to R_+$$

with similar properties. Now note that, the terms $y_{i,k}$ and z_l do not interact with each other if $l \neq t_i + k$. So, we may take such terms to be separable in ϕ^3. Also, if the terms $y_{i,k}$ and $y_{i,k'}$ for some $k \neq k'$ has scope for interaction, then we are implicitly assuming intertemporal mobility of income. Now suppose $y_{i,k} > z_{t_i+k}$ and $y_{i,k'} < z_{t_i+k'}$. Then the excess income, over subsistence, in period k may be transferred to period k' to ameliorate the extent of poverty in that period. So the excess income in period k would have some effect on the poverty calculation. But this violates (F). So, the terms $y_{i,k}$ and $y_{i,k'}$ will be separable in the arguments of ϕ^3. For similar reasons, the different z_{t_i+k} terms will also be separable. Thus, finally one can see that the only variables that may possibly interact among them are the pairs $(y*_{i,k}, z_{t_i+k})$, $k = 1, ..., l_i$. So, one can finally redefine ϕ^3 to arrive at the form of the aggregate poverty measure given by (2). The properties of the functions ϕ and f_l's, $l = 1, ..., l_i$, are inherited from that of ϕ^2. ■

Theorem 1 identifies a general class of measures that are useful for application to data and is simply parametrized. We will now look at the consequence of invoking alternative invariance assumptions on the poverty measure in (2). Here one can establish the following results.

Theorem 2: (a) The poverty measure given by (1) satisfies C, F, M, S, P, TR, D and SI if and only if for all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for life time income matrices $A = (Y_1, Y_2, ..., Y_n)$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n} \phi_R(R_i) \tag{3}$$

where $R_i = (r_{i,1}, r_{i,2}, ..., r_{i,l_i})$ with $r_{i,k} = \frac{y_{i,k}*}{z_{t_i+k}}$, for $k = 1, ..., l_i$. $\phi_R : \Omega \to R_+$ is continuous, decreasing and strictly convex in each of the arguments $r_{i,k}$, $k = 1, ..., l_i$ of R_i.

(b) The poverty measure given by (1) satisfies C, F, M, S, P, TR, D and TI if and only if for all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for life time income matrices $A = (Y_1, Y_2, ..., Y_n)$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n} \phi_B(B_i) \tag{4}$$

where $B_i = (b_{i,1}, b_{i,2}, ..., b_{i,l_i})$ with $b_{i,k} = z_{t_i+k} - y_{i,k}*$, for $k = 1, ..., l_i$. ϕ_B : $\Omega \to R_+$ is continuous, increasing and strictly convex in each of the arguments $b_{i,k}$, $k = 1, ..., l_i$ of B_i.

Proof : (a) Define u as a subsistence requirement vector all of whose entries equal one. Let $R = (R_1, ..., R_n)$ where R_i's are as defined in the statement. Now note that, due to SI, $P(A, T, S, n) = P(R, T, u, n)$

We now invoke the other axioms as in Theorem 1 to arrive at the form

$$\frac{1}{n}\sum_{i=1}^{n} \phi(f_1(r_{i,1}, 1), ..., f_{l_i}(r_{i,l_i}, 1))$$

with the desired properties. Now, this can be redefined as equation (3). This demonstrates the necessity part. Sufficiency can be easily verified by checking that the class of poverty measure given by (3) satisisfies all the assumptions.

(b) Again, if we define the subsistence requirement vector O, all of whose entries equal 0, we can similarly show that $P(A,T,S,n) = P(B,T,O,n)$ where $B = (B_1, ..., B_n)$.

We now invoke the other axioms as in Theorem 1 to arrive at the form

$$\frac{1}{n}\sum_{i=1}^{n}\phi(f_1(b_{i,1},0),...,f_{l_i}(b_{i,l_i},0))$$

with the desired properties. Now, this can be redefined as equation (4). Hence the necessity. Sufficiency is once again easy to check. ■

4.2 Personal Life Time Poverty

Given that we can formulate a reasonable aggregation rule for interpersonal level of poverty given by (3) or (4), depending on our value judgement about invariance laws, one should now try and find a solution to the other problem that we mentioned before; that of determining the life time poverty level of any peron. With the help of the intertemporal axioms, this we proceed to do in the following theorem.

Theorem 3: (a) The poverty measure given by (1) satisfies C, F, M, S, P, TR, D, SI, ITS and ITC if and only if for all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for life time income matrices $A = (Y_1, Y_2, ..., Y_n)$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n}G(\sum_{t=1}^{l_i}\psi_{l_i}^{R}(r_{i,t})) \tag{5}$$

where $G : R_+ \rightarrow R_+$ is continuous, increasing and convex. $\psi_{l_I}^{R} : R_+ \rightarrow R_+$ is continuous, decreasing and strictly convex.

(b) The poverty measure given by (1) satisfies C, F, M, S, P, TR, D, TI, ITS and ITC if and only if for all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for life time income matrices $A = (Y_1, Y_2, ..., Y_n)$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n}G(\sum_{t=1}^{l_i}\psi_{l_i}^{B}(b_{i,t})) \tag{6}$$

where $G : R_+ \to R_+$ is continuous, increasing and convex. $\psi^B_{l_I} : R_+ \to R_+$ is continuous, increasing and strictly convex.

Proof : (a) In view of theorem 2(a), we start with the form (3) and invoke ITC on $\phi_R(R_i)$, this is analogous to imposing D on interpersonal poverty measures and hence the form of $\phi_R(R_i)$ becomes

$$\sum_{t=1}^{l_i} \psi^R_{i,t}(r_{i,t}).$$

Now the axiom ITS imposes symmetry between the functions $\psi^R_{i,t}$ for each t. So the functions $\psi^R_{i,t}$ becomes independent on t and may only depend on l_i, which is individual specific parameter. Thus we finally arrive at the form $\psi^R_{l_i}(r_{i,t})$ for each $\psi^R_{i,t}(r_{i,t})$. Hence, the aggregate poverty measure now reduces to the form (5). Sufficiency can be easily verified by checking that the class of poverty measure given by (5) satisifies all the assumptions.

(b) The proof of this part is exactly similar to that of part (a), given theorem 2(b), and we omit the details. ■

The above theorem characterises a general class of aggregate life time poverty measures that satisfy the set of axioms put forth. The final choice for a practitioner may be any particular member of this class. This is a matter of value judgement and specific forms will follow from specific assumptions that are taken on the form of the functions $G(.)$ and $\psi(.)$ in theorem 3. Below we provide a few examples.

Simple illustrations of measures belonging to the class characterised by theorem 3 (a) is given by the following two examples. These would be reasonable measures if we assume some version of *intertemporal separability* on our life time poverty measure.

Example 1: Consider $G(x) = x$ and

$$\psi^R_{l_i}(r_{i,k}) = \frac{1}{l_i}(1 - (r_{i,k})^\delta).$$

So that the life time poverty measure becomes

$$P(A, T, S, n) = 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{1}{l_i} \sum_{k=1}^{l_i} (\frac{y_{i,k^*}}{z_{t_i+k}})^\delta, \tag{5.1}$$

where $0 < \delta < 1$ is some constant. For $\delta = 1$, this is analogous to average of income gap ratio.

Example 2: An analogue of the FGT measure is when we consider $G(x) = x$ again and

$$\psi_{l_i}^R(r_{i,k}) = \frac{1}{l_i}(1 - r_{i,k})^\alpha.$$

Then $P(.)$ becomes

$$P(A, T, S, n) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{l_i}\sum_{k=1}^{l_i}(\frac{z_{t_i+k} - y_{i,k}*}{z_{t_i+k}})^\alpha, \tag{5.2}$$

where $\alpha > 1$ is some constant.

Again, a simple example of measures belonging to the class characterised by theorem 3 (b) is given in example 3 below.

Example 3: $G(x) = x$ and

$$\psi_{l_i}^B(b_{i,k}) = \frac{1}{l_i}(b_{i,k})^\alpha.$$

Then,

$$P(A, T, S, n) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{l_i}\sum_{k=1}^{l_i}(z_{t_i+k} - y_{i,k}*)^\alpha, \tag{6.1}$$

for some constant $\alpha > 1$. For $\alpha = 1$, this can be written as

$$\frac{1}{n}\sum_{i=1}^{n}(\frac{1}{l_i}\sum_{k=1}^{l_i}(z_{t_i+k} - y_{i,k}*)) = \frac{1}{n}\sum_{i=1}^{n}(\frac{1}{l_i}\sum_{k=t_i+1}^{t_i+l_i} z_k - \bar{Y_i}*), \tag{6.2}$$

the average shortfall over a person's life time.

Note that these classes of measures closely resemble the aggregate deprivation measures discussed in **Mukherjee (2001)**. As poverty can be seen as a measure of deprivation arising dute to shortfall from a subsistence level, this proximity is quite natural.

5 Premature Mortality

Now that we have characterised the aggregate life time poverty measures for a set of individuals with a give life time and income profile, we will now try to tackle the issue of premature mortality, as indicated in section 2. That is, we will now ask how the length of life interacts with the income level and if so, how should we incorporate that into the poverty measure discussed above.

Consider two populations of n persons all born at the same date. In each, there are 2 poor persons with income y_1 and y_2 ($> y_1$) in each period of their lives and (n-2) persons who are non-poor throughout. All persons in population 1, all non-poor and the richer poor in population 2 lives for two periods. The poorest in population 2 live only for one period.

Now, lets compare traditional snapshot poverty levels in the two periods. In the first period, the income distributions are identical, so all usual measures will show the same level of poverty for both the populations. In the second period, poverty in population 2 will be lower according to any standard poverty measure *because the poorest person has died*! In fact, even if we use the life time poverty measure developed in the last section based on the life time income profiles of the persons in these two populations, poverty would still be unambiguously no higher in population 2 than in 1. Something must be wrong.

To solve the above anomaly, we proceed in the following manner. For any population, let us consider a normative length of life (say L); length of time each person in the population is expected to live upto. If she dies before the age L ($l_i < L$) then one has to take note of this premature death in the life time income profile itself. To facilitate this, one has to extend the income profile of person i from Y_i of length l_i to, say, $\widehat{Y_i}$ of length L. To achieve that, if $l_i < L$ then for person i, the income values are taken to be $E(y_{i,1}, ..., y_{i,l_i})$, $E : \Omega \rightarrow R_+$, in each of the periods $l_i + 1, ..., L$. Thus, this function gives us a proxy for the fictitious "income" of a dead individual. We assume that (i) E is increasing in each argument, (ii) $0 < E(x, .., x) \leq x$ for

$x > 0$ and (iii) $E(x_1, .., x_m) \geq z$ if $x_i \geq z_{t_x+i}$ for all i and for any $m \in Z_+$ (t_x being the birthdate of this person). These assumptions ensure that the proxy income is positive and less than the living income. That is, a person is better off alive[1]. Also, if a person was rich in all periods of his lifetime, then his proxy income will also be a nonpoor income. This assumption implies that a dead rich person's presence will not affect any poverty measure that focusses only on the incomes of the poor. Thus, if person i dies at age $3 < L$, then for this person we have the income profile $\widehat{Y}_i = (y_{i,1}, y_{i,2}, y_{i,3}, e, ..., e) \in R_+^L$ where $e = E(y_{i,1}, y_{i,2}, y_{i,3})$. We need to impose certain structure on the function $E(.)$ for it to be consistent with SI (or TI). This is (iv) $E(\lambda x_1, ..., \lambda x_m) = \lambda E(x_1, ..., x_m)$ for any $m \in Z_+$. That is, $E(.)$ is homogenous of degree one (or (iv) $E(aU + X) = a + E(X)$ for any $X \in \Omega$, U being a vector of ones of the same order as X and any $a \in R$ such that $aU + X \in \Omega$). Call the extended income profile for the population $\widehat{A} = \{\widehat{Y}_i, i = 1, ..., n\}$.

We now focus our attention on the possible alternative specifications of the function $E(.)$. Lets take a look back at our example of two populations discussed at the beginning of this section. If we assume that $E(y_1) = y_1$ as the second period proxy income for the poorest individual in population 2, then the poverty profiles will be identical for both the populations using the modified normative life time measures. That is, the poverty measure becomes insensitive to the fact that the poorest died early in population 2. Thus, the substitution of $E(y_1) = y_1$ (as done here) achieves a neutrality to death for our life time poverty measure, whereas earlier (without the correction for premature mortality) it was mortality averse (poverty reduces if poorest people die off). If we take $E(y_1) < y_1$, then this is consistent with the "better off alive" assumption. This way a penalty can be imposed on premature mortality.

So, we define the aggregate normative life time poverty measure defined on the

[1]One can also think of a situation where the death of a poor person may be considered as putting him out of his misery and hence improves the fictitious individual's well being. But this would immediately bring us back to the discussion involving the repugnant conclusion.

extended profile $\widehat{A}$ for the population under consideration by

$$P^e = P^e(\widehat{A}, T, S, n) : \Omega \times H \times \Omega \times z_+ \rightarrow R_+. \tag{7}$$

One can now invoke the assumptions laid out in section 3 on the extended income profiles $(\widehat{Y}_1, \widehat{Y}_2, ..., \widehat{Y}_n)$, in an analogous fashion to theorem 1 and 2, to arrive at similar personal life time poverty measures. Thus, individual level normative life time poverty may now be redefined as $\phi^e_R(\widehat{R}_i)$ or $\phi^e_B(\widehat{B}_i)$ (depending on whether SI or TI is invoked), where $\widehat{R}_i = (\widehat{r}_{i,1}, ..., \widehat{r}_{i,L})$ with $\widehat{r}_{i,t} = \widehat{y}_{i,t}*/z_{t_i+t}$ and $\widehat{B}_i = (\widehat{b}_{i,1}, ..., \widehat{b}_{i,L})$ with $\widehat{b}_{i,t} = z_{t_i+t} - \widehat{y}_{i,t}*$ for $t = 1, ..., L$ and $i = 1, ..., n$. Under the intertemporal axioms, ITS and ITC, one can now finally state the following modification of Theorem 3.

Theorem 4: (a) The normative life time poverty measure given by (7) satisfies C, F, M, S, P, TR, D, SI, ITS and ITC if and only if for all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for normative life time extended income matrices $\widehat{A} = (\widehat{Y_1}, \widehat{Y_2}, ..., \widehat{Y_n})$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n} G^e(\sum_{t=1}^{L} \psi^R_e(\widehat{r_{i,t}})) \tag{8}$$

where $G^e : R_+ \rightarrow R_+$ is continuous, increasing and convex. $\psi^R_e : R_+ \rightarrow R_+$ is continuous, decreasing and strictly convex.

(b) The normative life time poverty measure given by (7) satisfies C, F, M, S, P, TR, D, TI, ITS and ITC if and only if for all $n \in Z_+$, $S \in \Omega$, $T \in H$ and for normative life time extended income matrices $\widehat{A} = (\widehat{Y_1}, \widehat{Y_2}, ..., \widehat{Y_n})$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n} G^e(\sum_{t=1}^{L} \psi^B_e(\widehat{b_{i,t}})) \tag{9}$$

where $G^e : R_+ \rightarrow R_+$ is continuous, increasing and convex. $\psi^B_e : R_+ \rightarrow R_+$ is continuous, increasing and strictly convex.

Proof : Proof is immediate given that of theorem 3. We only need to consider the extended profiles and impose ITC and ITS. The function $\psi(.)$ is forced to be the same for all individuals as all the lengths of life now are equal. Hence we redefine it to be ψ_e^R or ψ_e^B in (a) and (b) respectively. ■

Example 4: (a) To illustrate the above theorem, consider the following form of the individual poverty indicator ψ_e^B given by $\psi_e^B(\widehat{b_{i,t}}) = \frac{1}{L}(\widehat{b_{i,t}})^\alpha$ and $G^e(x) = x$. The poverty measure would then be given by

$$P^e(\widehat{A}, T, S, n) = \frac{1}{nL}\sum_{i=1}^{n}(\sum_{t=1}^{L}(\widehat{b_{i,t}})^\alpha). \tag{9.1}$$

Similarly if we consider $\psi_e^R(\widehat{r_{i,t}}) = \frac{1}{L}(1 - \widehat{r_{i,t}})^\alpha$ and $G^e(x) = x$, we will obtain

$$P^e(\widehat{A}, T, S, n) = \frac{1}{nL}\sum_{i=1}^{n}(\sum_{t=1}^{L}(1 - \widehat{r_{i,t}})^\alpha). \tag{8.1}$$

This once again is an analogue of the FGT measure.

Also, our definition of the vector $\widehat{Y}_i$ implies that if a poor person dies before the normative age, then she will be considered as if she were poor during the periods subsequent to her death. As we will show, this has extremely interesting implications in the context of our measurement methodology.

To illustrate the difference of the measures we discuss with the traditional measures of poverty that do not take into account the mortality patterns of the population under consideration, consider the following example.

Example 5: Suppose the poverty measure we are using is analogous to income gap ratio. Consider two populations. Assume that the normative life time for both the population is 100. The subsistence requirement is \$ 1 each period.

Suppose in population 1, the percentage of poor is 20, all of whom live for 50 years and have \$ 1 income in each period of their lifetime. The proxy income, when they are dead, is taken to be \$ 0 (as a limiting value, for simplicity). Hence, the premature mortality corrected income gap ratio for this population will be given by $I_1 = 20\% \times$ (1 - $\frac{50}{100}$) = 10% Population 2 has 15% poor who live for 30 years and also earn \$ 1 each period with proxy income \$ 0. We similarly compute $I_2 = 15\% \times$ (1 - $\frac{30}{100}$) = 10.5%.

In this example, the traditional poverty gap measure and its usual life time version shows a lower value for population 2 but as the life span is much shorter, our mortality corrected measure shows a higher value for population 2 than for population 1.

Our methodology outlined above can be easily adapted for empirical purposes, as indicated in section 2. For a currently living population, we need to have data on birth dates (t_i), length of life (l_i) and income profiles (Y_i) of each member i in the population. Then one has to decide on a normative length of life L for this population to decide on the relevant set and a suitable proxy income function to arrive at the extended profiles, $\widehat{Y_i}$, for each (possibly hypothetical) person in the relevant set. The relevant set can be determined as discussed in section 2 above. Then, using equation (8.1) or (9.1), the aggregate poverty measure for the extended profile will now look like

$$\frac{1}{\sum_{i=1}^{n} I(L/l_i)} \sum_{i=1}^{n} I(L/l_i) \frac{1}{L} \sum_{t=1}^{L} (1 - \widehat{r_{i,t}})^{\alpha} \tag{8.2}$$

or

$$\frac{1}{\sum_{i=1}^{n} I(L/l_i)} \sum_{i=1}^{n} I(L/l_i) \frac{1}{L} \sum_{t=1}^{L} \widehat{b_{i,t}}^{\alpha}. \tag{9.2}$$

Note that this poverty measure will have all the desirable properties discussed in section 3. Now, if we take the income of each individual to be the same at each period of his/her life (say y_i) and take the subsistence requirements to be the same also (say z), then (8.2) and (9.2) reduces to

$$\frac{1}{\sum_{i=1}^{n} I(L/l_i)} \sum_{i=1}^{n} I(L/l_i) (\frac{z - y_i*}{z})^{\alpha} \tag{8.3}$$

and

$$\frac{1}{\sum_{i=1}^{n} I(L/l_i)} \sum_{i=1}^{n} I(L/l_i) (z - y_i*)^{\alpha} \tag{9.3}$$

respectively. Note that (8.3) is just the measure P^l discussed in section 2.

6 Conclusion

In the presence of premature mortality for the poorer sections of the population, standard snapshot poverty measures will show a decrease. To avoid this welfare measurement paradox, in this paper we develop and characterise a poverty measure based on the life time income profile of an individual. This measure does not exhibit such paradoxical behaviour but one can further modify this measure, defining it on a normative rather than actual life time of the individuals so that premature mortality of the poor actually effects the poverty measure positively. We characterise and illustrate such a measure here and indicate how to compute this measure in practise in a simple fashion. Choice of the normative length of life, L, is a crucial issue for this life time poverty measurement. The poverty ranking of a population may change with respect to other population groups if the value of L is changed, say from 80 to 70 years. Thus, a careful choice of L is an important element of this analysis.

We could have put forth a two-dimensional snapshot measure of well-being as a solution to this problem, with poverty and life expectancy of the population as the two determinants. But that would still have missed looking at the poor who are actually, but albeit prematurely, dead. This measure takes account of vital events like birth and death. But one could think of other important demographic events like immigration and emigration that affects the poverty status of a population. These issues are beyond the scope of our structure, but one might put forward a tentative solution as follows. An immigration or emigration results in a left or right truncated income profile for the relevant individual. For example, a profile like $(y_1, ..., y_k)$ or $(Y_{k+1}, ..., y_l)$ where l is the life time of the individual and $k < l$ is the year of transition. One may now complete these vectors as $(y_1, ..., y_k, z_{k+1}, ..., z_l)$ or $(z_1, ..., z_k, y_{k+1}, ..., y_l)$, where z_t is the subsistence requirement relevant for the period t of the individual's life time. A substitution of this type will make the poverty measure neutral to the income of this person in periods when she did not belong to the population. Now, these extended profiles may be used in place of the original truncated profiles and life time povery level for the individuals may be computed in the usual fashion.

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