Population Growth and Poverty Measurement

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Abstract

If the absolute number of poor people goes up, but the fraction of people in poverty comes down, has poverty gone up or gone down? The economist's instinct, framed by population replication axioms that undergird standard measures of poverty, is to say that in this case poverty has gone down. But this goes against the instinct of those who work directly with the poor, for whom the absolute numbers notion makes more sense as they cope with more poor on the streets or in the soup kitchens. This paper attempts to put these two conceptions of poverty into a common framework. Specifically, it presents an axiomatic development of a family of poverty measures without a population replication axiom. This family has an intuitive link to standard measures, but it also allows one or other of "the absolute numbers" or the "fraction in poverty" conception to be given greater weight by the choice of relevant parameters. We hope that this family will prove useful in empirical and policy work where it is important to give both views of poverty—the economist's and the practitioner's—their due.

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1. Introduction

The World Bank's calculations show that from 1987 to 1998, the number of people in the world surviving on less than two dollars a day increased from 2.5 billion to 2.8 billion. But the world's population was increasing sufficiently fast that the incidence of poverty, the percentage of people below the poverty line, fell from 61.0 percent to 56.1 percent.[1] Did world poverty fall or stay constant during this turbulent period of globalization? One answer to this question is to say that it is a non-question--the answer depends on what is meant by an increase in poverty. But this is precisely the point.

There appear to be two substantively different views of poverty increase (or decrease). One is associated with absolute numbers of the poor, the other with their number relative to the total population (or the "incidence" of poverty). The economist's instinct is to go with the latter. The instinct of those on the ground, for example those who have to face the absolute requirements of increased demands on soup kitchens or homeless shelters, is to think that poverty has gone up when the number of mouths to feed or beds to find goes up.

In the axiology of poverty measurement, which is where economists draw their instincts from, variously labeled axioms of population replication assure a neutrality with respect to population scale. These axioms basically argue the following: Take two identical societies and merge them to create a society with twice the population size. The poverty index in the merged society is the same as in the component societies even though the absolute number of the poor is twice as great, because the total population is twice as large as well.

The distinction between the two views of poverty is no mere technicality. As argued in Kanbur (2001), in Ghana between 1987 and 1991 the incidence of poverty came down by about one percentage point per year, while the absolute number of the poor increased because total population was growing by around two percentage points a year. The World

[1] See Tables 1 and 2.

Bank and the IMF trumpeted the first as a measure of the success of their recommended "structural adjustment" policies, while those in civil society who criticized these policies did so at least partly because as they looked around them they could see more poor people in the streets. The global figures on poverty reproduced in Tables 1 and 2 show many comparisons where change in absolute numbers and change in incidence move in opposite directions or, when they move in the same direction, do so at very different rates. For example, in South Asia the number of poor people increased by more than 180 million people, while the incidence of poverty fell by 2.7 percentage points. Even in East Asia excluding China, where both absolute numbers and incidence fell, the rate of fall was very different. Absolute numbers fell by 16 percent, while the incidence fell by 30 percent. In Sub-Saharan Africa, where both numbers and incidence rose, absolute numbers rose by 38 percent while incidence rose by a bare 2 percent. These contrasts raise questions about the recent U.N. "millennium target" for income poverty reduction—which has been specified in terms of the incidence of poverty rather than in terms of the absolute numbers of the poor.

The analysis in this paper puts the two conceptions on poverty measures—one, that the poverty measure should rise when the number of poor increases, and the other that the poverty measure should fall when, holding the number of poor constant, total population increases—into a common framework. Section 2 sets out the axiomatic framework and derives the basic characterization of a family of poverty measures without a population replication axiom. Section 3 discusses the basic result further, and shows how with different parameterizations the two different views can be given different weights within this family of measures. Section 4 concludes.

2. Framework and Basic Result

For a population of size *n*, the set of income distributions is given by R_+^n, the nonnegative orthant of the n-dimensional Euclidean space R^n. A typical element of R_+^n is

$x = (x_1, x_2, ..., x_n)$, where $x_i \geq 0$ is the income of person *i*. The set of all income distributions is $R_+ = \bigcup_{n \in N} R_+^n$, where *N* is the set of natural numbers.

A person i said to be poor if $x_i < z$, where $z \in R_{++}^1$ is the exogenously given poverty line, with R_{++}^1 being the strictly positive part of the real line. For any $n \in N$, $x \in R_+^n$, the set of poor persons is *S(x)* = $\{1 \leq i \leq n : x_i < z\}$ and the cardinality of *S(x)*, that is, the number of poor persons is *q(x)*. The censored income distribution associated with *x* is $x^* = (x_1^*, x_2^*, ..., x_n^*)$, where $x_i^* = z$ if $x_i \geq z$, and $x_i^* = x_i$ if $x_i < z$.

A poverty index is a real valued function of individual incomes, the population size and the poverty line. More precisely, a poverty index is a function P: $R_+ X R_{++}^1 X$ N $\rightarrow$ R^1. The restriction of P on $R_+^n X$ R$_{++}^1 X \{n\}$ is denoted by P^n, where $n \in N$ is arbitrary. For any $n \in N$, $x \in R_+^n$, $P^n(x; z; n)$ indicates the poverty level associated with the income distribution *x* distributed over the concerned population of size *n* and the poverty line *z*.

The poverty index is assumed to satisfy certain desirable properties. These are:

Focus (FOC): For all $n \in N, x, y \in R_+^n, z \in R_{++}^1$, if *S(x)* = *S(y)* and $x_i = y_i$ for all $i \in S(x)$, then $P^n(x; z; n) = P^n(y; z; n)$.

Monotonicity (MON): For all $n \in N, x, y \in R_+^n, z \in R_{++}^1$, if $x_j = y_j$ for all $j \neq i$, $i \in$ *S(x)*, $x_i > y_i$, then $P^n(x; z; n) < P^n(y; z; n)$.

Transfers Principle (TRP): For all $n \in N, x, y \in R_+^n, z \in R_{++}^1$, if $x_i = y_i$ for all $i \neq j$, *k* and $x_j > y_j \geq y_k > x_k, x_j - y_j = y_k - x_k$, for $k \in S(x)$ and $S(x) = S(y)$, then $P^n(x; z; n) > P^n(y; z; n)$.

Symmetry (SYM): For all $n \in N, x, y \in R_{+}^{n}, z \in R_{++}^{1}$, if y is a permutation of x , then $P^{n}(x;z;n) = P^{n}(y;z;n)$.

Increasingness in Subsistence Income (ISI): For all $n \in N, x \in R_{+}^{n}$, $P^{n}(x;z;n)$ is increasing in *z* over R_{++}^{1}.

Continuity (CON): For all $n \in N, z \in R_{++}^{1}$, $P^{n}(x;z;n)$ is continuous in $x \in R_{+}^{n}$.

Scale Invariance (SCI): For all $n \in N, x \in R_{+}^{n}$, $z \in R_{++}^{1}, P^{n}(x;z;n) = P^{n}(cx;cz;n)$, where $c>0$ is any scalar.

FOC says that the poverty index is independent of the incomes of nonpoor persons. According to MON , a reduction in the income of a poor must increase poverty. TRP demands that a transfer of income from a poor (j) to a richer poor (k) that does not change the set of poor persons increases poverty. SYM means that any characteristic other than income, e.g., the names of the individuals, is irrelevant to the measurement of poverty. Since given the income distribution, an increase in the poverty line makes the poor people more deprived in terms of income shortfalls from the poverty line, the poverty index should increase if the poverty line increases. This is what is demanded by ISI. CON ensures that minor observational errors in incomes will generate minor changes in the poverty index. SCI says that the poverty index is independent of the unit in which incomes and the poverty line are measured. (For further discussions on these properties, see Sen, 1976; Donaldson and Weymark, 1986; Cowell, 1988; Foster and Shorrocks, 1991 and Zheng, 1997).

We now adopt an axiom that combined with other axioms ensures additive separability of the poverty index.

Structural Separability (STS): There exists $E: R_{+} \to R^1$ and $w: N \to R^1$ such that for all $m, n \in N$, $x \in R_{+}^{m}$, $y \in R_{+}^{n}$, $z \in R_{++}^{1}$,

$$P^{m+n}(x, y; z; m+n) = Q\left(E^{m+n}(x, y); z; w(m+n)\right),$$
$$= A\left(Q(E^{m}(x); z; w(m)), Q(E^{n}(y), z; w(n))\right),$$

where $A: R^2 \to R^1$ is increasing in its arguments and the behaviour of E and Q are to be determined by P.

In STS we first assume a specific structure of the poverty index and then impose separability as an additional requirement. STS is similar to subgroup consistency of Foster and Shorrocks (1991), which requires overall poverty for a population partitioned into subgroups to increase if poverty in one or more subgroups increases and stays constant in others. The function *E* may be regarded as an aggregate deprivation function. Deprivation may be measured in terms of relative or absolute shortfall of each income from all higher incomes. We can also view it in terms of divergence of each income in the corresponding censored income distribution from the poverty line.

We now state a theorem which characterizes the family of poverty indices which satisfy the above axioms. Note, in particular, that the axiom set does not include any of the variants of the "population replication" axiom.

Theorem 1: A poverty index $P: R_{+} X\ R_{++}^{1} X\ \ N \to R^1$ satisfies FOC, MON, TRP, CON, SCI, and STS if and only if for all $m \in N$, $x \in R_{+}^{m}$, $z \in R_{++}^{1}$, $P^{m}(x; z; m)$ is ordinally equivalent to

$$\sum_{i=1}^{m} p\left(\frac{x_i^*}{z}\right) - \alpha w(m), \qquad (1)$$

where $p: [0,1] \to R^1$ is continuous, decreasing, strictly convex, *w* is increasing and α is a constant.

Proof: In Appendix.

The monotonicity principle we have used in theorem 1 was suggested by Sen(1976). However, the index in (1) satisfies a stronger monotonicity condition, which requires poverty to decrease if there is an increase in a poor person's income (see Donaldson and Weymark, 1986). This latter condition includes the possibility that the beneficiary of the income increase may become rich. Analogously, the index satisfies the Sen(1976) version of the transfer axiom, a stronger requirement than the TRP considered by Donaldson and Weymark (1986). The Sen version of the transfer axiom requires poverty to increase under a transfer of income from a poor to anyone richer. Note that in this case if the two persons involved in the transfer are poor, then the transfer may make the recipient rich so that the set of poor persons changes.

To relate theorem 1 with existing results, let us denote the first term of (1) by T^m. Foster and Shorrocks (1991) showed that all subgroup consistent poverty indices must be of the form $F\left(T^m/m\right)$, where F is continuous and increasing. Clearly, there are some important differences between the class isolated in theorem 1 and the Foster-Shorrocks family. While the latter is population replication invariant, the former is not. Another source of difference is the appearance of the term $\alpha w(m)$ in (1), which enables us to consider different views on poverty change under population growth. Specifically, notice that the first term of (1) is an “aggregate” (not normalized by total population) version of standard poverty measures that emerge from settings where population replication axioms are imposed. The second term depends purely on total population and is impact depends on the choice of the parameter α and the function *w(m)*. These two terms allow us to see the different implications of population growth for the measure of poverty. The next section discusses these implications.

3. Discussion

In the rest of the paper we will assume, for simplicity, that p satisfies the normalization condition $p(1)=0$. We will now show how aggregate counterparts to different population replication invariant subgroup consistent indices can be derived as particular cases of (1). For this we make the assumption that $\alpha = 0$. As a first example, let p(t) = t^{δ}, where for MON and TRP to hold we need δ >1. The underlying index becomes

$$P_{\delta}^{m}(x;z;m) = \sum_{i \in S(x)} \left(1 - \frac{x_i}{z}\right)^{\delta} . \tag{2}$$

P_{δ} is the aggregate version of the Foster - Greer - Thorbecke (1984) index. For $0 < \delta \leq 1$, the index satisfies NON but not TRP. As $\delta \to 0$, $P_{\delta}^{m} \to q(x)$, the absolute number of poor. For $\delta = 1$, P_{δ}^{m} becomes the aggregate income gap ratio of the poor, which can be rewritten as

$$P_{\delta}^{m} = q(x) I(x) , \tag{3}$$

where $I(x) = \sum_{i \in S(x)} (1 - x_i / z) / q(x)$ is the income gap ratio of the poor. On the other hand, if $\delta = 2$, the index becomes

$$P_{\delta}^{m} = q(x)\left[(I(x))^2 + (1 - I(x))^2 C^2(x)\right] , \tag{4}$$

where *C (x)* is the coefficient of variation of the income distribution of the poor. Thus, over the income distributions with the same number of poor and the same mean income of the poor, the ranking of distributions generated by P_{δ} (for δ =2) is same as that produced by C. Note that the number of nonpoor incomes and their distribution are immaterial for this ranking. It is easy to check that an increase in the value of δ >2 makes the index more sensitive to transfers lower down the scale.

An alternative of interest arises from the specification $p(t) = 1 - t^{c}$, where 0<c<1 ensures that MON and TRP are fulfilled. The corresponding index is given by

$$P_c^m(x;z;m) = \sum_{i \in S(x)} \left(1 - \left(\frac{x_i}{z}\right)^c\right), \tag{5}$$

which is the aggregate version of the Chakravarty (1983) index. For any 0<c<1, a transfer of income from a poor to a rich increases P_c^m by a larger amount the poor the donor is. If we assume c>1, then TRP is violated but MON is satisfied and as $c \to \infty, P_c^m \to q(x)$ .For c=1, P_c^m coincides with *q(x)I(x)*.

As a last example, assuming that all incomes are positive, let us suppose that *p(t) = - logt*. This generates the aggregate form of the Watts (1968) poverty index

$$P_w^m(x;z;m) = \sum_{i \in S(x)} \log\left(\frac{z}{x_i}\right). \tag{6}$$

P_w is more sensitive to transfers at the lower end of the distribution.

Setting $\alpha \le 0$ in (1), we note that, given positivity of *w(m)* , an increase in the number of poor increases poverty. Next, assuming positivity of both α and *w(m*), we note that the poverty index in (1) decreases unambiguously as the total population increases, keeping the number of poor constant. This shows how the two views concerning poverty change as a consequence of change in the population size have been incorporated in a general structure.

Kundu and Smith (1983) demonstrated that there does not exist any poverty index that meets the Sen (1976) version of the transfer principle and the two above conceptions on poverty change because of population growth. The main difference between the Kundu-Smith formulation and ours is that we do not impose the two population growth criteria at the outset, rather we derive the two views separately as implications of our general formula (1).

Finally, we want to examine poverty behaviour when the absolute number of poor increases but at a slower rate than the overall population. Assume again that both α and *w(m)* are positive. Since we will be dealing with continuous changes, we denote the first term in (1) by T , poor population size by q and assume that T,q and *w(m)* are continuously differentiable.

Let us now consider fractional replications of the poor and overall populations at different rates $\frac{dq}{q}$ and $\frac{dm}{m}$, where $\frac{dq}{q} < \frac{dm}{m}$, with $q+dq$ ($m+dm$) being the replicated population size of the poor (overall community). Hence the rate of growth of the poverty aggregate T will be $\frac{dT}{T} = \frac{dq}{q} < \frac{dm}{m}$. To understand this more explicitly, suppose that the income distribution of the poor is (4, 4, 5, 5, 6, 6) and the poverty line is 10. Assuming that dq=3, the replicated income distribution of the poor is (4, 4, 4, 5, 5, 5, 6, 6, 6). It is then easy to see that $\frac{dq}{q} = \frac{dT}{T} = 1/2$.

We denote the rates dT/T and dm/m by μ and λ respectively. Then the rate $d(T/m)/T/m$ becomes $(\mu - \lambda)$. Hence by our earlier discussion, $\mu - \lambda < 0$. We can now write $d(T - \alpha w(m))$, the change in the poverty index $T - \alpha w(m)$, as $dT - \alpha\, w'(m)\, dm$= $\mu T - \alpha\lambda m w'(m)$.

All population replication invariant poverty indices will fall under a change of the type considered. But an aggregate function of the form T will increase. For the overall poverty index (1) to increase even allowing for a discounting for population growth rate, the index should fulfill the inequality $0 < \mu\mathrm{T} - \alpha\lambda\mathrm{mw}'(\mathrm{m}) < \mathrm{dT}$.

Proposition 2: (i) $\mu\mathrm{T} - \alpha\lambda\mathrm{mw}'(\mathrm{m}) < \mathrm{dT}$ holds if and only if *w(m)* is increasing in *m*. (ii) $0 < \mu T - \alpha\lambda m w'(m)$ holds if $w'(m) < 1/m$ and α is small.

Proof:

(i) Since $\mu\mathrm{T} = d\mathrm{T},\ \mu\mathrm{T} - \alpha\lambda m w'(m) < d\mathrm{T}$ holds if and only if $\alpha\lambda m w'(m) > 0$. Given positivity of α, λ and m, $\alpha\lambda m w'(m)$ is positive if and only if $w'(m) > 0$, that is, *w* is increasing.

(ii) Since $\mu T = d\mathrm{T}\ and \lambda m = dm,\ \mu T - \alpha\lambda m w'(m) > 0$ if and only $\alpha w'(m) < d\mathrm{T} / dm$. Suppose $dT = \in > 0$. Then under the assumption that the population growth rate is normal $(dm < m)$, we get $\frac{dT}{dm} > \frac{\in}{m}$. Hence it is sufficient to consider that $\alpha w'(m) < \in / m$. We can now choose $\alpha = \in > 0$ in the above inequality to get $w'(m) < 1/m$.

This completes the proof of the proposition .

As an illustrative example, we can take $w(m) = log(1+m)$. It is easy to check that proposition 2 holds for this specification of *w*. Part (i) of proposition 2 justifies increasingness of *w*. That is, increasingness of *w* is necessary and sufficient to ensure that poverty will increase under population growth but the aggregate function *T* will increase faster than when *T* is accompanied by the absolute population component $\alpha w(m)$. Next, part (ii) of the proposition shows that a sufficient condition for the index $T - \alpha w(m)$ to rise is that the marginal $w'(m)$ is small (given that $\alpha > 0$ is small).

It can thus be seen how different combinations of the *w(m)* function and different values of α combine to generate different responses of the poverty measure to population growth. In particular, consider the specific poverty index in (1) and set $w(m) = log(1+m)$. We then have an index that is an intuitive combination of the "aggregative" version of standard poverty indices (that allow aversion to depth of poverty as necessary), and a correction factor that depends on the parameter α and which tends to pull the index back to the "population normalized" view. We have shown that for this index, as α varies one or the other view of poverty is given greater weight. For negative, zero and small positive values of α, replications of the poor population increase poverty even when accompanied by replications of the non-poor population such that total population grows faster than the number of the poor. But for large enough values of α, such a combination of population and poor replication will reduce the poverty index.

4. Conclusion

Population replication axioms are now so much a part of the axiology of poverty measurement that economists take them on board without much thought. They have a certain appeal, they are certainly convenient, and help to generate families of poverty measures that we have all become familiar with. But, as we have argued in the introduction, they impose a structure on poverty measures that do not necessarily conform to the intuitions and instincts of those who deal with the daily realities of poor people's lives. We have shown, however, that appealing poverty measures can indeed be derived without population replication axioms. These measures relate intuitively to standard measures, and are tractable and applicable in empirical and policy work. They also allow, through choice of parametrizations, for different weights to be given to the "absolute numbers" versus the "fraction in poverty" views. Given these properties, we hope that this family of measures will prove their worth in empirical and policy work.

Appendix

Proof of Theorem 1

By SCI, $P^{m+n}\left(\frac{x}{z},\frac{y}{z};1;\mathrm{m}+\mathrm{n}\right)=P^{m+n}(x,y;z;m+n)$, which in view of STS becomes $Q\left(E^{m+n}\left(\frac{x}{z},\frac{y}{z}\right);1\quad w(m+n)\right)$. Rewrite the last expression as $H\left(E^{m+n}\left(\frac{x}{z},\frac{y}{z}\right);\ w(m+n\right)$. Applying STS once again we can show that $P^{m+n}(x,y;z;m+n)\ =A\left(H(E^{m}(\frac{x}{z});w(m)),H\left(E^{n}(\frac{y}{z});w(n)\right)\right)$

Therefore,

$$A\left(H\left(E^{m}(\frac{x}{z});\ w(m)\right),H\left(E^{n}(\frac{y}{z});\ w(n)\right)\right)=H\left(E^{m+n}\left(\frac{x}{z},\frac{y}{z}\right);\ w(m+n)\right). \tag{7}$$

FOC implies that $P^{m+n}(x,y;z;m+n)\ =\ P^{m+n}(x^{*},y^{*};z;\ m+n)$. Hence (7), under FOC, becomes

$$A\left(H\left(E^{m}(\frac{x^{*}}{z});\ w(m)\right),H\left(E^{n}(\frac{y^{*}}{z});\ w(n)\right)\right)=H\left(E^{m+n}\left(\frac{x^{*}}{z},\frac{y^{*}}{z}\right);\ w(m+n)\right). \tag{8}$$

Putting n=1, m=1 in (8) we get

$$A\left(H\left(E^{1}(\frac{x_1^{*}}{z});\ w(1)\right),H\left(E^{1}(\frac{y_1^{*}}{z});\ w(1)\right)\right)=H\left(E^{2}\left(\frac{x_1^{*}}{z},\frac{y_1^{*}}{z}\right);\ w(2)\right). \tag{9}$$

Given continuity of P^{m}, the general solution to the functional equation (9) is given by

$$A\ \ (u,v)=\ f^{-1}\left(f(u)+f(v)\right), \tag{10}$$

$$E^2(r,s) = p^{-1}\left(p(r) + p(s)\right), \tag{11}$$

$$H(l,w(m)) = f^{-1}(p(l) - \alpha w(m)), \tag{12}$$

where f is an arbitrary continuous, strictly increasing function, p is a real valued function, w is increasing on N and α is a constant (Aczel, 1966, theorem 3 and Corollary 4, pp. 314-315).

By repeated use of (11) for any $b \in R_+^m$, we get

$$E^m(b) = p^{-1}\left(\sum_{i=1}^{m} p(b_i)\right) \quad , \tag{13}$$

where $m \in N$ is arbitrary.

Note that l , the first argument of H in (12) is the value of the function E^m at x^*/z for some $x \in R_+^m$, $m \in N$ and $z \in R_{++}^1$ and H is a representation of the poverty index. Therefore, in the presence of STS, SCI and FOC, in view of (12) and (13), we have

$$P^m(x;z;m) = f^{-1}\left(p\left(p^{-1}\sum_{i=1}^{m} p\left(\frac{x^*}{z}\right)\right) - \alpha w(m) \right) = f^{-1}\left(\sum_{i=1}^{m} p\left(\frac{x_i^*}{z}\right) - a w(m) \right). \tag{14}$$

Since f is increasing, f^{-1} is so. Clearly, the domain of p in (14) is [0, 1]. By increasingness of f^{-1}, P^m in (14) satisfies MON only if p is decreasing. A similar argument shows that for TRP to hold we need strict convexity of p.

We will now demonstrate that p is continuous. Suppose to the contrary that given $x_i (i = 1,\ 2,..,k-1,\ k+1,..,m)$, p is discontinuous at x_k^*/z. Then for $x = (x_i,..,x_{k-1}, x_k, x_{k+1},..,x_m)$ and $z \in R_{++}^1$

$$P^m(x;z;m) = f^{-1}\left(\sum_{\substack{i=1 \\ i \neq k}}^{m} p\left(\frac{x_i^*}{z}\right) + p\left(\frac{x_k^*}{z}\right) - \alpha w(m) \right)$$

$$= f^{-1}\left(p(\frac{x_k^*}{z}) + B \right) \text{(say)}. \tag{15}$$

Define $y_i = x_i$ if $i \neq k$ and $y_k = x_k + \sigma$ where $\sigma < 0$. Then

$$P^m(y; z; m) = f^{-1}\left(p(\frac{y_k^*}{z}) + B \right). \tag{16}$$

Now discontinuity of p at $\frac{x_k^*}{z}$ shows that there exists $\in > 0$ such that for some $\delta > 0$, $\left| \frac{x_k^*}{z} - \frac{y_k^*}{z} \right| < \delta$ implies $\left| p(\frac{x_k^*}{z}) - p(\frac{y_k^*}{z}) \right| > \in$. But continuity of P^m demands that for $\in' > 0$, $\left| \frac{x_k^*}{z} - \frac{y_k^x}{z} \right| < \delta$ implies $\left| P^m(x; z; n) - P^m(y; z; n) \right| < \in'$, that is, $\left| f^{-1}\left(p(\frac{x_k^*}{z}) + B \right) - f^{-1}\left((p(\frac{y_k^*}{z}) + B \right) \right| < \in'$. Thus, we have $\left| f^{-1}(t) - f^{-1}(t') \right| < \in'$ but $|t - t'| > \in$, where $t(t') = p\left(\frac{x^*}{z} \right) + B \left(p(\frac{y_k^*}{z}) + B \right)$. This can happen only if f^{-1} is a constant function, a contradiction to the assumption that f^{-1} is increasing. Using similar arguments we can demonstrate that f^{-1} is also continuous.

This completes the necessity part of the proof of the theorem. The sufficiency is easy to verify .

Note that in proving the theorem we did not assume SYM. However, the poverty index in (1) satisfies SYM, because E^m in (13) satisfies it.

Table 1

Population Living On Less Than $2 Per Day, 1987 and 1998

Regions	**Number of people living on less than $2 day (millions)**	
	1987	**1998**
East Asia and the Pacific	1,052.3	884.9
(excluding China)	299.9	252.1
Eastern Europe and Central Asia	16.3	98.2
Latin America and the Caribbean	147.6	159.0
Middle East and North Africa	65.1	85.4
South Asia	911.0	1,094.6
Sub-Saharan Africa	356.6	489.3
Total	**2549.0**	**2,811.5**
(excluding China)	1,796.6	2,178.7

Source: World Bank, 2001 Poverty Update
http://www.worldbank.org/html/extdr/pb/pbpoverty.htm

Table 2

Percent Of People Living On Less Than $2 per day, 1987, and 1998

Regions	**Percent of people living on less than $2 day**	
	1987	**1998**
East Asia and the Pacific	67.0	48.7
(excluding China)	62.9	44.3
Eastern Europe and Central Asia	3.6	20.7
Latin America and the Caribbean	35.5	31.7
Middle East and North Africa	30.0	29.9
South Asia	86.3	83.9
Sub-Saharan Africa	76.5	78.0
Total	**61.0**	**56.1**
(excluding China)	58.2	57.9

Source: World Bank, 2001 Poverty Update
http://www.worldbank.org/html/extdr/pb/pbpoverty.htm

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