Spatial referencing

An overview

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Learning instructions

Learning activities:

- Literature: ITC Core textbook, Chapter 3.1 on Spatial Referencing.
- Website: http://kartoweb.itc.nl/geometrics
- Exercise: Spatial referencing (ArcMap10)

Questions: Blackboard Discussion Board
Main objectives

- Understand the relevance and actual use of reference surfaces, coordinate systems, and coordinate transformations in mapping.

- Describe and differentiate between coordinate systems and map projections.

- Grasp the logic of map projection equations and the principles of transforming maps from one projection system to another.
Contents

- Spatial reference surfaces and datums
  - The Geoid - vertical (height) datum
  - The Ellipsoid - horizontal (geodetic) datum
  - Local and global datums

- Map projections
  - Classification of map projections
  - Map projection selection
  - Map coordinate systems (e.g. UTM)

- Coordinate transformations
Earth to Map

The Earth: a complex shape

Ellipsoid (best fitting) → Independent handling of horizontal and vertical

Geodetic (or horizontal) datum

Plane

Cone

Cylinder

Mapping surface

Azimuthal map projection

Conical map projection

Cylindric map projection

mean sea level

Height (or vertical) datum
Reference surfaces for mapping

The Earth

The ellipsoid

The Geoid

Independent handling of horizontal and vertical
Spatial reference surface

- The Geoid
The Geoid - Vertical (height) datum

The Earth

The Geoid

Global Sea Level

Amsterdam Tide-gauge

Benchmark of the levelling network

neatest benchmark of the levelling network

point whose height is to be determined

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Starting from Mean Sea Level (MSL) points, the heights (H) of points on the Earth can be measured using a technique known as geodetic leveling.

\[ H = \text{Orthometric height} \]

Tide-gauge benchmark (zero height)

mean sea level

\[ H = \text{Orthometric height} \]
Vertical datums

Every country (or group of countries) has its own Mean Sea Level - its own vertical (height) datum.

MSL of Belgium is 2.34m lower than MSL of The Netherlands.
At what height do we live?

http://www.ahn.nl/postcodetool

Elevation data are related to Amsterdam Zero (N.A.P.)
GPS height versus N.A.P. height

GPS Reference Stations

Enschede
Station ID 0550

Position and Height
52° 13'25" N
6° 53'10" E
107,51 m

Location
This station is located on the roof of the International Institute for Geo-Information Science and Earth Observation (ITC) in the centre of Enschede.

This station is sending RTCM 18/19 RTK data and storing static data 24 hours a day. This station is also a part of the GlobalNET and receives the GLONASS signal.

\[ H_{\text{NAP}} = h_{\text{WGS84}} - 44\text{m (N)} - 27\text{m (ITC building)} = 37\text{m} \]
Ellipsoidal height versus Orthometric height

Ellipsoidal height

topography

h

b

giocentre

ellipsoid

Orthometric height (height above the Geoid)

topography

H

H

giocentre

geoid
The earth's surface, and the geoid and a reference ellipsoid used to approximate it. The geoidal undulation (N) is the separation between the geoid and an ellipsoid. It varies globally between ±110 m.

H = Orthometric height
h = Ellipsoidal height
N = Geoidal separation (undulation)
Trends in mapping: global vertical datums

Satellite gravity missions (e.g. GOCE) make it possible to determine a global vertical datum with centimetres accuracy.
Spatial reference surface
- The Ellipsoid
Typical values of the parameters for an ellipsoid:

\[ a = 6378137.0 \text{ m} \quad b = 6356752.31 \text{ m} \]
\[ f = 1/298.26 \quad e = 0.0818187 \]

**Flattening:**
\[ f = (a-b)/a \]

**Eccentricity:**
\[ e^2 = (a^2 - b^2)/a^2 \]
Geographic coordinates \((\varphi, \lambda)\)

- \(\varphi = 52^\circ 9' 18.62''\)
- \(\lambda = 5^\circ 23' 13.93''\)

\(\varphi\) (phi) = latitude angle

\(\lambda\) (lambda) = longitude angle

- \(h = 67\text{m}\)
Local and global ellipsoids/datums

The ellipsoid is chosen in such way that it best fits the surface of the area of interest (the country)

- e.g. WGS84
- e.g. Amersfoort (Bessel)
- Region of best fit
- Ellipsoid globally best fitting to the geoid
- Ellipsoid regionally best fitting to the geoid

The geoid
Local datum system of the Netherlands

Ellipsoid

Bessel ellipsoid

Geodetic datum

National triangulation network

Local datum system of the Netherlands

\[ \begin{align*}
\phi_A &= 52° 13' 26.2'' N \\
\lambda_A &= 06° 53' 32.1'' E \\
X_{RD} &= 257776.47 m \\
Y_{RD} &= 471588.14 m
\end{align*} \]

Geodetic datum

\[ \begin{align*}
\phi_A &= 52° 9' 22.2'' N \\
\lambda_A &= 5° 23' 15.5'' E \\
X_{RD} &= 155000 m \\
Y_{RD} &= 463000 m
\end{align*} \]
Local datum systems

Countries (or regions) use their own datum system to make accurate maps.

<table>
<thead>
<tr>
<th>Datum</th>
<th>Ellipsoid</th>
<th>Datum shift (m)*</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(Dx, Dy, Dz)</td>
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<tr>
<td>Alaska (NAD-27)</td>
<td>Clarke 1866</td>
<td>-5, 135, 172</td>
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<td>-4, 154, 178</td>
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<tr>
<td>Bermuda 1957</td>
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<tr>
<td>Central America (NAD-27)</td>
<td>Clarke 1866</td>
<td>0, 125, 194</td>
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<tr>
<td>Bellevue (IGN)</td>
<td>Hayford</td>
<td>-127, -769, 472</td>
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<tr>
<td>Campolnchaupe</td>
<td>Hayford</td>
<td>-148, 136, 90</td>
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<tr>
<td>Hong Kong 1963</td>
<td>Hayford</td>
<td>-156, -271, -189</td>
<td></td>
</tr>
<tr>
<td>Iran</td>
<td>Hayford</td>
<td>-117, -132, -164</td>
<td></td>
</tr>
</tbody>
</table>

* positions compared to WGS84
Local and global ellipsoids/datums

The ellipsoid is chosen in such way that it best fits the surface of the area of interest (the country).

- *e.g. WGS84*
- *e.g. Amersfoort (Bessel)*

Ellipsoid globally best fitting to the geoid

Ellipsoid regionally best fitting to the geoid

Region of best fit

The geoid
International Terrestrial Reference System (ITRS)

Global reference system for global surveying and mapping. The system uses the GRS80 (or WGS84) ellipsoid.
Global ellipsoids and datums to approximate the earth-as-a-whole - with the aid of satellites - are becoming more in use (e.g. WGS84, ITRF, ETRS89).

Changing or re-adjustment of local ellipsoids and datums is taking place in many countries.
Map projections
Map projection principle

2D Cartesian coordinates \((X,Y)\)

- \(\phi = 55^\circ 00' 00''\ N\)
- \(\lambda = 45^\circ 00' 00''\ W\)

\(x = 280,000\ m\)
\(y = 310,000\ m\)
A map projection is a mathematical function by which 2D Geographic coordinates $(\phi, \lambda)$ are transformed into 2D Cartesian map coordinates $(x, y)$

\[ (x, y) = f(\phi, \lambda). \] \hspace{1cm} \text{Forward equation}

\[ (\phi, \lambda) = f^{-1}(x, y). \] \hspace{1cm} \text{Inverse equation}
Map projection equations (example)

Map projection equations for the Mercator projection (spherical assumption)

Forward mapping equation:

\[ x = R(\lambda - \lambda_0) \]

\[ y = R(\ln(\tan(\frac{\pi}{4} + \frac{\phi}{2}))) \]

Inverse mapping equation:

\[ \phi = \frac{\pi}{2} - 2 \arctan(e^{\frac{-y}{R}}) \]

\[ \lambda = \frac{x}{R} + \lambda_0 \]
Mercator projection
Conformal cylindrical projection
Scale distortions on the Mercator projection

Area distortions are significant towards the polar regions. Greenland appears to be larger but is only one-eighth the size of South America.
Any map projection is associated with scale distortions. The amount and kind of distortions depend on the type of map projection.
Map projection properties

Conformal
Angles and shapes are correctly represented (locally)

Equivalent (or equal-area)
Areas are correctly represented

Equidistant
Distances from 1 or 2 points or along certain lines are correctly represented
Cylindrical equal-area projection

Areas are correctly represented
Equidistant cylindrical projection
(also called Plate Carrée)

Equidistant along the meridians
Robinson projection
Pseudo-Cylindrical

Neither conformal nor equal-area (both shape and area are reasonably well preserved)
Classification of map projections

- Class
  - Cylindrical
  - Conical
  - Azimuthal

- Secant or tangent projection plane

- Property
  - Equivalent (or equal-area)
  - Equidistant
  - Conformal

- Aspect (orientation)
  - Normal
  - Oblique
  - Transverse

Secant projection planes

Transverse cylindrical
Oblique conical
Classification of map projections

Conformal cylindrical projection with a transverse cylinder and secant projection plane (e.g. Universal Transverse Mercator)
Classification of map projections

CONICAL

Normal  Oblique  Transverse

Secant  Tangent

Equivalent  Equidistant  Conformal

Conformal conical projection with a normal cone and tangent projection plane (e.g. Lambert conformal conic)
Classification of map projections

- **AZIMUTHAL**
  - Polar
  - Oblique
  - Equatorial
    - Secant
    - Tangent
  - Equivalent
  - Equidistant
  - Conformal

Conformal azimuthal projection with a tangent polar projection Plane (e.g. Universal Polar Stereographic)
Selection of a Map projection (I)

Normal *cylindrical projections* are typically used to map the World in its entirety. *Conical projections* are often used to map the different continents, while the normal *azimuthal projection* may be used to map the polar areas.

Also consider the **shape of the area** to be mapped:
Selection of a Map projection (II)

Conformal
Maps which require measuring angles (e.g. aeronautical charts, topographic maps)

Equivalent (or equal-area)
Maps which require measuring areas (e.g. distribution maps)

Equidistant
Maps which require reasonable area and angle distortions (e.g. several thematic maps)
Selection of a Map projection (III)

The position (and orientation) of the projection plane is optimal when the projection plane is located along the main axis of the area to be mapped, or when the projection centre coincides with centre of the area.
Dutch map coordinate system

Projection: Stereographic
Geodetic datum: Amersfoort (Bessel ellipsoid)
Universal Transverse Mercator

International Standard
Universal Transverse Mercator projection

Conformal Cylindrical (transverse secant) projection
UTM Mapping zones

Longitudinal zone of 6°

84°N

80°S
UTM Mapping zones

Equator

Central Meridian

0° 6°

Greenwich

1 29 30 31 32
Two adjacent UTM zones

Zone 10
Central Meridian

Zone Junction

Zone 11
Central Meridian

E = 500,000 m

SF = 0.9996

40 km overlap

126° 123° 120° 117° 114° W
UTM grid on maps

Topographic map of the Netherlands (scale 1:25,000)
Coordinate transformations
Position City hall of Enschede

Position in Geographic coordinates:

- $\phi_{\text{Amersfoort}} = 52^\circ 13' 26.2''N$  $\lambda_{\text{Amersfoort}} = 6^\circ 53' 32.1'' E$
  
  ($\phi_{\text{Bessel}} = 52.223944^\circ N$  $\lambda_{\text{Bessel}} = 6.8922489^\circ E$)

- $\phi_{\text{ETRS89}} = 52^\circ 13' 22.6''N$  $\lambda_{\text{ETRS89}} = 6^\circ 53' 29.7'' E$

- $\phi_{\text{WGS84}} = 52^\circ 13' 22.6''N$  $\lambda_{\text{WGS84}} = 6^\circ 53' 29.7'' E$

Position in Map (plane rectangular) coordinates:

- $X_{\text{Dutch RD}} = 257790.12m$  $Y_{\text{Dutch RD}} = 471607.17m$ (Old RD1918)

- $X_{\text{Dutch RD}} = 257776.47m$  $Y_{\text{Dutch RD}} = 471588.14m$ (New RD)

- $X_{\text{UTM31}} = 765872.57m$  $Y_{\text{UTM31}} = 5793185.04m$

- $X_{\text{UTM32}} = 356065.01 m$  $Y_{\text{UTM32}} = 5788133.6m$
Map projection change
Using projection equations

Inverse equations
Cartesian reference coordinate system I

Forward equations
Cartesian reference coordinate system II

Projection A
P (244,249)

Projection B
P (208,309)

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Map projection change
Including a datum transformation

Inverse equations
Forward equations
Datum shifts
Coordinate transformations (overview)
Projection change using a 2D Cartesian transformation

Conformal, Affine or Polynomial transformation

Control point

Projection unknown

Projection B

The unknown coordinate system is related to a known coordinate system on the basis of a set of known points.
Application: Image Rectification (I)

True position

Measured position

Error vector

Point 1

Point 2

Point 3

Point 4

Link Table

<table>
<thead>
<tr>
<th>Link</th>
<th>X Source</th>
<th>Y Source</th>
<th>X Map</th>
<th>Y Map</th>
<th>Residual</th>
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</tbody>
</table>

Total RMS Error: 1.24382
Application: Image Rectification (II)

Georeferencing

Image Rectification (geocoding)
Application: Matching data layers
Thank you!