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Wealth Accounting, Exhaustible Resources and Social Welfare

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Abstract The empirical literature on natural resource accounting uses methods which implicitly or explicitly entail measuring changes in total resource asset value when an exhaustible resource is depleted. In contrast, the growth theoretic literature on saving, social welfare and sustainable development is built upon a central finding, that the change in real wealth (as measured by net or 'genuine' saving) is proportional to the change in social welfare. We show that the change in total wealth exceeds the change in real wealth in optimal and non-optimal models of resource-extracting economies. This suggests that the change in social welfare is over-estimated when the change in total resource asset value is used as the measure of depletion. A simple empirical exercise, using World Bank data on 'adjusted net saving', reinforces the results from theory.

Keywords Environmental accounting · Exhaustible resources · Genuine saving · Social welfare

JEL Classification Q01 · Q03

1 Introduction

This paper explores an apparent disconnection between the theory of wealth, social welfare and sustainable development, and the empirical practice of 'green' national accounting. The problem revolves around the link between changes in wealth and changes in social welfare when one of the assets of the economy is an exhaustible resource.

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The literature on wealth accounting and social welfare (Hamilton and Clemens 1999; Dasgupta and Mäler 2000; Asheim and Buchholz 2004, and Pezzey 2004, for example) shows that it is the change in *real* wealth (as measured by net or 'genuine' saving) which has welfare significance, because the change in real wealth is proportional to the change in social welfare. The literature on rules for sustainable development (including Hartwick 1977; Dixit et al. 1980; Hamilton and Hartwick 2005; Hamilton and Withagen 2007) has a similar point of departure—different policy rules for sustainability are possible, but they are all built around a measure of the change in real wealth.

In contrast, in the literature on exhaustible resource accounting there is a tradition of estimating the depreciation of exhaustible resource assets when resources are extracted using the change in *total* asset value (including capital or holding gains) as the appropriate measure of depletion. This is evident in Hartwick and Hagemann (1993) who show that one of the primary papers in the literature on green national accounting, El Serafy (1989), is implicitly valuing resource depletion as the change in total asset value. Similarly, much of the empirical literature on adjusting the national accounts to reflect depletion of natural resources (including the UN system of Integrated of Environmental and Economic Accounting—see United Nations 2003) has used methods which reflect changes in total wealth.

Davis and Moore (2000) provide a fairly recent attempt to develop the theory and practice of accounting for exhaustible resources. Their aim is show that shortcut methods such as valuing resource assets at the current total rental rate (price minus the average cost of extraction) may provide biased estimates of asset values. They derive a measure of the value of an exhaustible resource asset when extraction costs are an increasing function of both resource extraction and cumulative resource extraction, and provide empirical estimates to show how this differs from shortcut valuation methods. In line with much of the literature, however, they define the value of depletion as the change in the total asset value.

Cairns (2000) gives a careful theoretical analysis of a mine as an enterprise for the extraction of exhaustible resources. This implies that the measure of depreciation for the enterprise combines both depreciation of the produced capital employed in extraction and the value of depletion of the resource stock. Because the resource price is exogenously given at the enterprise level, the measure of depletion of the resource (which, again, is derived as the change in total asset value) includes the value of future resource price changes—this parallels the results shown in Sect. 3 below. Cairns (2002) shows that util-valued assets (produced capital, exhaustible resources, and labor) sum to total social welfare in a simple economy, and again identifies depletion of the exhaustible resource stock with the change in total asset value, including capital gains.

The motivation for the current paper is to clarify what questions the green national accounting literature is, or is not, answering in the context of exhaustible resources. The literature clearly provides the answer to one important question: What is the change in total asset value when a resource is extracted? But it does not answer another question which should be profoundly important to policy makers: How much has social welfare changed when this resource is extracted?

The paper analyzes these issues incrementally. We first present an optimal closed economy model characterized by constant returns to scale, exhaustible resources and costly resource extraction. Next, this model is extended to the case of an optimal small resource-exporting economy. This is followed by an example of a non-optimal resource extracting economy. Finally we look at practical asset accounting, measuring both the change in total asset value and the change in real asset value using data from the World Bank. Our result is quite general: in all cases we find that the change in total wealth is greater than the change in real wealth, suggesting that the change in social welfare is being overestimated in much of the resource accounting literature dealing with exhaustible resources.

2 Wealth and Social Welfare in an Optimal Closed Economy

This section derives some basic results concerning the change in total and real wealth along the development path in an optimal closed economy. In order to compare changes in total and real wealth it turns out to be useful to derive alternative measures of income for this simple economy.

Assume a Dasgupta-Heal type economy with a finite stock of resource S which is extracted at rate R, and where production depends on the capital stock and flow of resources, i.e. $F = F(K, R)$. We assume optimal growth, where the present value of utility $U(C)$ is maximized for a fixed pure rate of time preference ρ. There are constant returns to scale in production, but not in resource extraction costs $f(R)$. The basic accounting identities for the economy are:

$$\dot{K} = F(K, R) - C - f(R)$$

$$S = \int_t^\infty R(s)\, \mathrm{d}s$$

$$\dot{S} = -R$$

Social welfare V is the present value of utility,

$$V \equiv \int_t^\infty U(C(s))\, \mathrm{e}^{-\rho(s-t)}\, \mathrm{d}s.$$

This is maximized over time under the optimal program. Supporting this optimum, the Hotelling rule for efficient resource pricing is,

$$\frac{\frac{\mathrm{d}}{\mathrm{dt}}\left(F_R - f'\right)}{F_R - f'} = F_K$$

Genuine saving is defined as,

$$G \equiv \dot{K} + \left(F_R - f'\right)\dot{S} = \dot{K} - \left(F_R - f'\right)R. \tag{1}$$

Genuine saving is a measure of the change in real wealth and, owing to optimality, values depletion at the marginal rental rate. From Hamilton and Clemens (1999) we know that on the optimal growth path genuine saving is equal to the dollar-valued change in social welfare,

$$G = \frac{1}{U_C}\dot{V}. \tag{2}$$

Total wealth is the sum of produced capital and the value of the natural resource stock,

$$W \equiv K + N = K + \int_t^\infty \left(F_R(s)\, R(s) - f(R(s))\right) \cdot \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} ds. \tag{3}$$

Note that the value of the resource stock N is given by the present value of total rents, not marginal rents (this is because infra-marginal rents also accrue to the owner of the resource).

This implies that there is no straightforward way to define a price p such that $N = pS$, which makes it difficult to compare the change in real natural resource asset value, $p\dot{S}$, with the change in total asset value, $p\dot{S} + \dot{p}S$. The solution to this problem, as noted earlier, is to compare two alternative measures of income.

The first step in measuring the difference between total and real changes in wealth is to use the assumed constant returns to scale to derive,

$$\dot{K} = F - C - f = F_K K + F_R R - C - f$$

The change in total wealth can therefore be derived from expression (3) as,

$$\begin{aligned} \dot{W} &= \dot{K} + \dot{N} \\ &= \dot{K} + F_K N - F_R R + f \\ &= F_K K - C + F_K N \\ &= F_K W - C \end{aligned} \tag{4}$$

$\dot{W}$ implicitly includes capital gains.

Next we relate the change in consumption to the level and change in genuine saving by applying the Hotelling rule (cf. Hamilton and Hartwick 2005),

$$\begin{aligned} \dot{C} &= \dot{F} - \ddot{K} - f'\dot{R} - \frac{\mathrm{d}}{\mathrm{d}t}\left(F_R - f'\right)R + \frac{\mathrm{d}}{\mathrm{d}t}\left(F_R - f'\right)R \\ &= F_K\dot{K} - F_K\left(F_R - f'\right)R - \ddot{K} + \left(F_R - f'\right)\dot{R} + \frac{\mathrm{d}}{\mathrm{d}t}\left(F_R - f'\right)R \\ &= F_K G - \dot{G} \end{aligned} \tag{5}$$

We can now calculate the difference between changes in total and real wealth by deriving our two Hicksian measures of income,[1]

$$NNP_1 = C + \dot{W}$$

and

$$NNP_2 = C + G.$$

Using expression (4) we see that NNP_1 is just equal to the return on total wealth,

$$NNP_1 = C + \dot{W} = F_K W. \tag{6}$$

From expressions (2) and (5) we can conclude that the growth in NNP_2 is proportional to the change in social welfare (cf. Asheim and Weitzman 2001),

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(NNP_2\right) = \frac{F_K}{U_C}\dot{V}.$$

If income equals consumption plus the change in total wealth, then we have a neatly self-contained and intuitive theory of income and wealth—income equals the return on total wealth—but there is no obvious link to social welfare. If instead income equals consumption plus the change in real wealth, then growth in income is proportional to the change in social welfare.

We derive the difference between $\dot{W}$ and G as follows. Expression (5) can be integrated to yield (cf. Sefton and Weale 1996)

$$C + G = \int_t^\infty F_K(s)\, C(s) \cdot \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau}\mathrm{d}s, \tag{7}$$

[1] Asheim (2000) refers to these as wealth-equivalent national income and green national income respectively.

while expression (4) can be integrated to yield,

$$W = \int_t^\infty C(s) \cdot \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s. \tag{8}$$

Subtracting expression (7) from expression (6), and applying expression (8), we therefore have,

$$\dot{W} - G = \int_t^\infty (F_K(t) - F_K(s))\, C(s) \cdot \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s. \tag{9}$$

Assuming that produced capital accumulates everywhere along the optimal path and that there are declining marginal returns to capital, it follows that the change in total wealth will be greater than the change in real wealth.[2] The change in total wealth overstates the impact of saving effort on social welfare.

3 The Optimal Small Resource-Exporting Economy

Next we explore the difference between changes in total wealth and changes in real wealth when the preceding model is extended to the case of a small resource-exporting economy. The key difference between this and the preceding model is the existence of exogenously changing international interest rates and resource prices. The details of this model are presented in Hamilton and Bolt (2004). The basic changes to the preceding model include splitting resource extraction between domestic and export uses,

$$R = R_\mathrm{d} + R_x,$$

and introducing the accounting relationship for net foreign assets A (shown below). Imports are denoted M, while the interest rate on foreign assets is r and the international resource price is p (r and p follow exogenously given paths $\{r(t)\}$ and $\{p(t)\}$). The optimization problem therefore becomes,

$$\max V = \int_t^\infty U(C(s))\, \mathrm{e}^{-\rho(s-t)} \mathrm{d}s \qquad \text{subject to:}$$

$$\begin{aligned} \dot{K} &= F(K, R_\mathrm{d}) + M - C - f(R) \\ \dot{S} &= -R = -(R_\mathrm{d} + R_x) \\ \dot{A} &= rA + pR_x - M \end{aligned}$$

Efficiency on the optimal path requires that $F_{R_\mathrm{d}} = p$ and $F_K = r$. The shadow price of the resource is given by $n \equiv F_{R_\mathrm{d}} - f'$ which follows the Hotelling rule, $\dot{n}/n = F_K$. Net saving Z is defined as $Z \equiv \dot{K} + \dot{A} - nR$, while genuine saving G is given by,

$$\dot{V} = U_C \left(Z + \int_t^\infty \dot{r}(s)\, A(s)\, \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s + \int_t^\infty \dot{p}(s)\, R_x(s)\, \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s \right) \equiv U_C G$$

Genuine saving therefore includes the present value of future capital gains on foreign assets and resource exports.

[2] This result is derived in a more general setting in Asheim (2000).

Hamilton and Bolt (2004) go on to show that,

$$\dot{C} + \dot{Z} = F_K Z + \dot{r} A + \dot{p} R_x,$$

from which it is simple to show that,

$$\dot{C} = F_K G - \dot{G}. \tag{10}$$

This is just the equivalent of expression (5). Total wealth for this model is given by,

$$W \equiv K + A + N = K + \int_0^t (p(s) R_x(s) - M(s)) \cdot \mathrm{e}^{\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s$$
$$+ \int_t^\infty (p(s) R(s) - f(R(s))) \cdot \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s$$

Assuming constant returns to scale, we can derive

$$\begin{aligned} \dot{W} &= \dot{K} + \dot{A} + \dot{N} \\ &= F - C + M - f(R) + F_K A + pR_x - M + F_K N + f(R) - pR \\ &= F_K K + pR_\mathrm{d} - C + F_K A + pR_x + F_K N - pR \\ &= F_K W - C \end{aligned} \tag{11}$$

This is the equivalent of expression (4). The analogues to expressions (7–9) follow immediately.

Assuming that produced capital is accumulating over the optimal path and that there are declining marginal returns to produced capital, we therefore arrive at precisely the same conclusions for this model as in the closed economy model.

4 A Non-Optimal Resource Producer

We now pose the same questions about changes in total versus real wealth in a non-optimal economy where the social planner is not aiming to maximize social welfare. Given the dominant role of the El Serafy (1989) approach in the resource accounting literature, it is useful to explore asset accounting in what might be termed an 'El Serafy economy.'[3] The salient features of this economy are that (i) resources are exhausted in finite time, and (ii) total rents on extraction are constant in each period up to the point of exhaustion.

We again assume a Dasgupta-Heal type economy with a finite stock of resource $S(t)$ which is extracted at rate $R(t)$, and where production depends on the capital stock $K(t)$ and flow of resources, $F = F(K, R)$. Resource extraction costs are given by $f(R(t))$. Declining marginal returns to each factor are assumed, as well as constant returns to scale, while resources are not essential for production. Utility is given by $U(C(t))$, and the pure rate of time preference ρ is constant. Resource extraction occurs over a finite horizon from time t to T, and it is efficient in the sense that the resource stock is fully depleted at time T.

For a given initial endowment at time t of $K(t)$ and $S(t)$, the social planner chooses paths for resource extraction $\{R(t)\}$ and gross saving $\{\dot{K}(t)\}$ as the control variables

[3] Strictly speaking, the El Serafy approach requires that both unit total rents and the quantity extracted are constant up to the point of exhaustion. We relax this assumption slightly in this section, by assuming only that total rents on extraction are constant, but apply it strictly in Sect. 5.

for the economy. Consumption is then determined residually. By assumption, total rents $\bar{\pi} = F_R R - f(R)$ are held constant in each time period up to the point of exhaustion.

The basic accounting relationships are:

$$C = F(K, R) - \dot{K} - f(R), \quad S = \int_t^T R(s)\,\mathrm{d}s, \quad \dot{S} = -R.$$

The consequence of constant total rents is,

$$\dot{\bar{\pi}} = \dot{F}_R R + F_R \dot{R} - f' \dot{R} = 0,$$

which implies that,

$$F_R \dot{R} - f' \dot{R} = -\dot{F}_R R, \tag{12}$$

and therefore,

$$\frac{\dot{F}_R}{F_R} = \left(\frac{f'}{F_R} - 1\right) \cdot \frac{\dot{R}}{R}.$$

If extraction is profitable at the margin ($F_R > f'$) and extraction is falling, then resource prices will be rising.

We use expression (12) to derive the expression for the change in consumption,

$$\dot{C} = F_K \dot{K} + F_R \dot{R} - \ddot{K} - f' \dot{R} = F_K \dot{K} - \ddot{K} - \dot{F}_R R. \tag{13}$$

Now define genuine saving as,

$$G \equiv \dot{K} - \int_t^\infty \dot{F}_R(s)\, R(s)\, \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s.$$

Here the second term is the present value of capital gains on resource extraction. The value of these capital gains drops to 0 once the resource exhaustion time T is reached—beyond this point genuine saving is just equal to investment in produced capital.

The change in genuine saving is given by,

$$\dot{G} = \ddot{K} - F_K \int_t^\infty \dot{F}_R(s)\, R(s)\, \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s + \dot{F}_R R,$$

so that, using expression (13),

$$\begin{aligned} F_K G - \dot{G} &= F_K \dot{K} - F_K \int_t^\infty \dot{F}_R(s)\, R(s)\, \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s - \ddot{K} \\ &\quad + F_K \int_t^\infty \dot{F}_R(s)\, R(s)\, \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s - \dot{F}_R R \\ &= \dot{C} \end{aligned} \tag{14}$$

Turning to the wealth accounts for this economy, the value of the resource stock N is,

$$N = \bar{\pi} \int_t^\infty \mathrm{e}^{-\int_t^s F_K(\tau)\mathrm{d}\tau} \mathrm{d}s,$$

and total wealth W is given by $W \equiv K + N$. Constant returns to scale yields,

$$\dot{K} = F - C - f = F_K K + F_R R - C - f = F_K K + \bar{\pi} - C.$$

The change in total wealth is therefore given by,

$$\dot{W} = \dot{K} + \dot{N} = F_K K + \bar{\pi} - C + F_K N - \bar{\pi} = F_K W - C. \tag{15}$$

Expressions (14) and (15) mirror expressions (5) and (4) respectively, so the analogues of expressions (7)–(9) follow immediately. If produced capital is accumulating along the non-optimal path, then the assumed declining marginal returns to produced capital lead to the same conclusion as in the previous two models: the change in total wealth exceeds genuine saving (at least during the period when resources are being extracted).

The linkage between genuine saving and changes in social welfare is less direct for non-optimal economies than for optimal ones. Because the Ramsey rule ($F_K = \rho - \dot{U}_C/U_C$) cannot be assumed to hold, there is no direct linkage between instantaneous genuine saving and the instantaneous change in social welfare, as is the case in an optimal economy. However, expression (14) for the change in consumption can support a generalized Hartwick Rule in this economy (Hamilton and Hartwick 2005; Hamilton and Withagen 2007). As long as there are feasible paths $\{R(t)\}$ and $\{\dot{K}(t)\}$ such that genuine saving is positive and growing at a rate less than the interest rate at each point in time, then expression (14) implies that consumption and utility will be rising everywhere along the development path.

Because the pure rate of time preference ρ is constant, it follows that,

$$V = \int_t^{\infty} U(C(s))e^{-\rho(s-t)}ds \quad \Rightarrow \quad \dot{V} = \int_t^{\infty} \dot{U}(C(s))e^{-\rho(s-t)}ds.$$

Therefore as Hamilton and Withagen (2007) show for competitive economies, if in this economy genuine saving is positive and growing at a rate less than the interest rate over an unbounded interval $[t, \infty)$, then social welfare will be everywhere increasing over this interval.

5 Practical Wealth Accounting

In practical wealth accounting for exhaustible resources it is typically assumed that the resource stock $S(t)$ is exhausted over some period $T - t$, that extraction follows a given path $\{R(t)\}$, and that each unit of the resource has total rental value following a given path $\{n(t)\}$. For a fixed interest rate r the asset value of the resource stock is given by,

$$N(t) = \int_t^T n(s)\, R(s)\, e^{-r(s-t)}ds. \tag{16}$$

If we take the time derivative of this value and rearrange terms we get what Hartwick and Hagemann (1993) call the 'fundamental equation of asset equilibrium':

$$\dot{N} + nR = rN.$$

In equilibrium, the sum of the change in the total value of the stock plus current rents must equal the opportunity cost of holding the asset.

Good models of the future path for extraction or unit rents are often lacking and so, as a practical matter, the wealth accountant falls back on simplifying assumptions. A typical assumption is that both unit total rents and per-period extraction are constant over the life of the resource (this is generally termed the simple present value or El Serafy approach).

We can formalize this assumption using the approach of Dasgupta and Mäler (2000). For production technology $F(K, R)$ with constant returns to scale and a non-essential resource input, the social planner chooses an *allocation mechanism* which ensures that per-period extraction $\bar{R}$ is constant, as is unit rent $\bar{n}$. For extraction cost $f(R)$ the unit total rent is $\bar{n} = F_R - f\left(\bar{R}\right)/\bar{R}$, while the (fixed) interest rate is given by $r = F_K$.

The basic accounting identities for the economy are,

$$F(K, R) = C + \dot{K} + f(R), \quad S(t) = (T - t)\,\bar{R} \text{ and } \dot{S}(t) = -\bar{R},$$

and the value of the resource stock is given by,

$$N(t) = \bar{n}\bar{R}\int_t^T e^{-r(s-t)}ds. \tag{17}$$

Defining $W \equiv K + N$, constant returns to scale implies that we can derive the analogue to expression (4) for this economy,

$$\begin{aligned}\dot{W} &= \dot{K} + rN - \bar{n}\bar{R}\\ &= F_K K + F_R\bar{R} - C - f\left(\bar{R}\right) + rN - \bar{n}\bar{R}.\\ &= rW - C\end{aligned}$$

This follows from the assumption that $\bar{n} = F_R - f\left(\bar{R}\right)/\bar{R}$ and $r = F_K$.

Solving this differential equation for W yields the analogue to expression (8),

$$W \equiv K + N = \int_t^\infty C(s)\cdot e^{-r(s-t)}ds.$$

Total wealth is just equal to social welfare measured in dollars. Now, following Dasgupta and Mäler (2000), we can define the *accounting price* $p(t)$ for the resource asset as,

$$p(t) \equiv \frac{\partial W(t)}{\partial S(t)} = \frac{\partial N(t)}{\partial S(t)} = \frac{\bar{n}}{T-t}\cdot\int_t^T e^{-r(s-t)}ds = \frac{\bar{n}}{T-t}\cdot\frac{1}{r}\left(1 - e^{-r(T-t)}\right). \tag{18}$$

This price measures the marginal contribution of the resource asset to social welfare, which implies that the change in social welfare associated with extracting quantity $\bar{R}$ of the resource is measured by $p\cdot\dot{S}$, the change in the real value of the stock. Since $S(t) = (T-t)\,\bar{R}$, expression (17) implies that,

$$N = p\cdot S.$$

Table 1 Alternative values of depletion for $\bar{n} = 10$ and $\bar{R} = 100$

	$T - t = 10$		$T - t = 25$		$T - t = 50$	
	$p \cdot \dot{S}$	$\dot{N}$	$p \cdot \dot{S}$	$\dot{N}$	$p \cdot \dot{S}$	$\dot{N}$
$r = 4\%$	824	670	632	368	432	135
$r = 10\%$	632	368	367	82	199	7

Source: Authors' calculations

For this simple model we want to compare the change in total asset value $\dot{N}$ with the change in real asset value $p \cdot \dot{S}$. From expression (17) we have,

$$\begin{aligned} \dot{N}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\bar{n}\bar{R}\int_t^T \mathrm{e}^{-r(s-t)}\mathrm{d}s\right) \\ &= rN - \bar{n}\bar{R} \\ &= r\bar{n}\bar{R}\int_t^T \mathrm{e}^{-r(s-t)}\mathrm{d}s - \bar{n}\bar{R} \\ &= \bar{n}\bar{R}\left(1 - \mathrm{e}^{-r(T-t)}\right) - \bar{n}\bar{R} \\ &= -\bar{n}\bar{R}\cdot \mathrm{e}^{-r(T-t)} \end{aligned} \tag{19}$$

The final equality in expression (19) is just El Serafy's 'user cost' measured in continuous time, and it is straightforward to show that $\dot{N}(t) = p\dot{S} + \dot{p}S = -\bar{n}\bar{R}\cdot \mathrm{e}^{-r(T-t)}$. The fundamental equation of asset equilibrium appears as an intermediate step in this derivation—this parallels Hartwick and Hagemann (1993), who show that the El Serafy method is implicitly measuring the change in total asset value. This can be compared to the change in real asset value using the accounting price,

$$p \cdot \dot{S} = -p \cdot \bar{R} = -\frac{\bar{n}\bar{R}}{T-t}\cdot\frac{1}{r}\left(1 - \mathrm{e}^{-r(T-t)}\right) = -\frac{N}{T-t}. \tag{20}$$

The change in the real value of the resource stock is equal to the total value of the stock divided by the remaining reserve lifetime. Dividing expression (20) by expression (19) it is clear that $p \cdot \dot{S} > \dot{N}$, and that both values converge to $-\bar{n}\bar{R}$ as the interest rate or the reserve life approach 0. Table 1 gives some numerical examples.

For a 'typical' situation with a 25 year reserve life and a social discount rate of 4%, the value of depletion using the change in real asset value is about 70% larger than that suggested by the El Serafy formula. For the same discount rate and a 50 year reserve life the real change in asset value is roughly 3 times the change in total asset value.

Table 2 presents depletion and genuine saving estimates for a range of major petroleum and natural gas producers, using data on 'adjusted net saving' from World Bank (2007).[4] In this analysis resource lifetimes are capped at 25 years, so this can be interpreted as a generational

[4] The figures for adjusted net saving published by the World Bank are calculated as net national saving plus education expenditure, minus resource depletion, minus pollution damages. Owing to sparse data on resource lifetimes, the Bank equates the value of resource depletion to the total rent on extraction. It is straightforward to show that this measures the change in real wealth if it can be assumed that $\dot{n}/n = r$ at each point in time. However, this is a strong assumption (a Hotelling-like rule for unit total rents) and it is reasonable to conclude that the Bank methodology over-estimates the value of resource depletion, especially for long-lived resource endowments. This point is noted in the World Bank's documentation.

Table 2 Depletion and genuine saving % of GNI in selected countries, 2005

Country name	Oil	NG	Depletion		Genuine saving	
	$T-t$	$T-t$	ES (%)	RW (%)	ES (%)	RW (%)
Algeria	19	25	22	35	25	12
Angola	12	25	47	61	−27	−42
Argentina	10	19	7	9	10	8
Azerbaijan	25	21	28	48	13	−7
Bolivia	21	25	13	23	3	−6
Brazil	17	25	2	3	7	6
Canada	25	10	3	4	12	10
Ecuador	23	25	10	17	6	−1
Egypt	16	25	10	15	6	1
Gabon	25	25	21	37	21	6
Indonesia	13	25	5	7	13	11
Iran	25	25	18	32	17	4
Kazakhstan	20	25	22	35	3	−10
Malaysia	11	25	10	14	15	11
Mexico	11	25	6	8	9	7
Nigeria	25	25	22	38	1	−15
Norway	8	22	13	17	17	13
Russian Federation	18	25	16	25	13	4
Syria	15	25	18	26	−8	−16
Thailand	7	19	3	4	21	20
Trinidad and Tobago	16	25	23	37	2	−12
Tunisia	12	25	3	5	14	13
Uzbekistan	14	25	35	57	1	−21
Venezuela	25	25	16	28	18	6
Vietnam	4	25	12	13	19	17

Key parameters: r: 4%; maximum $(T-t)$: 25
NG: Natural gas; $T-t$: Remaining reserve lifetime; ES: Change in total wealth method (El Serafy); RW: Change in real wealth method. Depletion and genuine saving figures are % of GNI
Source: Derived from World Bank (2007)

wealth account. The depletion figures indicate that the change in real resource wealth (in this case the sum of petroleum and natural gas depletion) is substantially larger than the change in total resource wealth calculated by the El Serafy method. A key result is that seven major producers of petroleum and natural gas—Azerbaijan, Bolivia, Ecuador, Kazakhstan, Nigeria, Trinidad and Tobago, and Uzbekistan—have positive genuine saving by the El Serafy method, but negative genuine saving when calculated as the change in real wealth.

6 Discussion and Conclusions

Atkinson and Hamilton (2007) approach the question of practical asset accounting from a different perspective than what is presented here. On the one hand they question whether using marginal rental values is useful (as optimal extraction models suggest), simply because real world mines or oil wells may not be optimally managed. On the other hand they argue that the lack of *any* attempt to maximize the value of the resource asset inherent in the El Serafy approach is likely to lead to artificially low asset values and therefore correspondingly low values for the depletion of the asset. Here we have taken an approach that is more firmly rooted in the theory of wealth and social welfare.

The theoretical models of optimal and non-optimal economies with an exhaustible resource presented in this paper suggest that the change in total wealth should exceed the change in real wealth at each point in time. Our exploration of a simple empirical model of an exhaustible resource deposit shows that this is true in practice—commonly used methods such as the El Serafy formula are in fact measuring the change in total value of the resource asset when resources are extracted, and this is lower than the change in the value of the stock in real terms. Since standard approaches to natural resource accounting are measuring net saving as $\dot{K} + \dot{N}$, this implies that the change in social welfare for the economy is being overstated when these approaches are employed.

These results strongly suggest that standard practices in the natural resource accounting literature (particularly valuing resource depletion according to the El Serafy formula) are measuring net saving, and therefore the change in social welfare, with an upward bias. The empirical estimates reported here show that this bias can be sufficient to change the sign of genuine saving from positive to negative in many resource-extracting countries.

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