

ACCOUNTING FOR DEPLETION OF NATURAL ASSETS IN THE 1993 SNA

Introduction

The 1993 System of National Accounts marks an important step forward in accounting for natural assets which are recorded as economic assets in the opening and closing balance sheets. This entails that the System must provide a complete accounting for changes in their values between the beginning and the end of the accounting period. In the 1993 SNA, an important distinction is introduced between acquisitions and disposals of assets through transactions and increases or decreases due to "other" factors, including "other volume" as well as price changes.

Nevertheless, the accounting for natural assets in the 1993 SNA is not fully worked out because there was not sufficient time in the later stages of the revision process to reach a consensus about the optimal method of recording and valuing the depletion of subsoil assets. Originally, it was hoped to finish the revision by the 1991 Statistical Commission. Although this proved not to be possible, it was generally agreed that the process must be completed by the 1993 Commission, whether or not a consensus could be reached on every problem. Inevitably, therefore, a few issues remained which could not be resolved to everyone's satisfaction because further research was needed. Among the unresolved issues placed on the research agenda (see "Perspectives on the 1993 SNA," section B, p.xliii) was environmental accounting. Although this paper suggests changes to the SNA, it should be viewed in this context as an attempt to settle some unfinished business. It starts by examining how subsoil assets should be valued in the balance sheets of the System and how, and where, changes in the valuation should be recorded in the system. Subsoil assets are used as an example which can be generalised to non-renewable resources. Later the extension to renewable resources is examined.

Recording of environmental assets in the accumulation accounts

The 1993 SNA recommends including in balance sheets environmental assets which are the subject of economic transactions. For those under human control, such as cultivated livestock and forests, the entries in the accounts are completely consistent with (other) produced fixed capital such as machinery and construction. Additions to the stock of the assets as a result of production are recorded as increase in inventories or as fixed capital formation; decreases in the stock are treated as withdrawals from inventories or as consumption of fixed capital. Changes in the value of the assets due to price variation and changes in their volume due to natural or man-made disasters are recorded in the other changes in assets account.

For natural assets that are not produced under human control, there is not a complete parallel in the recommended recording in the accumulation accounts. Taking sub-soil assets as the initial example, all changes in both the volume and value between the opening and closing balance sheet levels are recorded in the other changes in assets account. While this is appropriate for price revaluations and the effects of disasters, depletion is a result of economic transactions and so should not be included in that account.

In principle a balance sheet is drawn up using the information valid at the date of the balance sheet concerning stocks of assets but also prices, technologies, interest rates and general expectation held on the date of the balance sheet. Between January 1st and December 31st of any year, all these factors may change. The structure of the accumulation accounts is to separate these changes into those that are due to economic transactions (for inclusion in the capital or financial account) from the others that appear in the other changes in assets account.

The other changes in assets account in the 1993 SNA has two sub-accounts: the other changes in volume of assets account and the revaluation account which records both nominal and real holding gains.

Other changes in volume of assets account

This account refers to changes in the volume of assets held by institutional units that are not attributable to actual or imputed transactions. In the case of subsoil assets it has one particularly important function, namely recording the "economic appearance" or "economic disappearance" of these assets. The subsoil assets recorded in the balance sheets are those that have positive prices because they are capable of bringing economic benefits to their owners. New subsoil assets can therefore make their first appearance in balance sheets for two kinds of reasons:

- 1) the discovery of new exploitable subsoil assets;
- 2) changes in technology, relative prices, or other factors that convert existing known deposits of zero previous value into commercially exploitable deposits with a positive value.

The other changes in volume of assets is also used to record the disappearance of physical assets other than those used up in production or consumption: for example, the destruction of assets by natural disasters such as floods or earthquakes that may render subsoil assets unusable. Subsoil assets that become "obsolete" because of unexpected changes in technology, relative prices or other factors can also disappear from the balance sheets.

The revaluation account

This account records nominal and real holding gains or losses on assets. Such gains or losses may accrue to owners of subsoil assets or holders of leases to extract subsoil assets.

It is shown in the 1993 SNA that if the total value of all transactions and other changes in the volume of a particular type of asset are subtracted from the difference between the closing and opening balance sheet figures, the remainder must equal the total value of the nominal holding gains or losses accruing on quantities of the asset held at any time during the accounting period. Thus, holding gains or losses can be calculated residually, if necessary, from balance sheet data and information on transactions and other volume changes.

Depletion

Depletion is defined as the decline during the accounting period in the present value of the subsoil asset to the owner purely as a result of the extraction of the asset. That is, depletion is the result of a production process and should be recorded as a transaction. It has to be measured with reference to a constant set of prices in order to separate the value of depletion, as such, from any accompanying holding

gains or losses. In order to remove the effect of changing prices from the changing value of any asset, it may be convenient to view transactions as if they all took place at the average prices within the period as this would lead to the same total value for the transactions as is actually recorded. The same approach may be adopted with depletion. Thus, in order to be consistent with the valuation of transactions, the present values of the stock of the subsoil asset at the beginning and end of the period used in the calculation of depletion should both be based on the average price of the asset within the period.

After taking account of any acquisitions or disposals or other volume changes, the difference between depletion calculated this way and the actual difference between the opening and closing balance sheet values (calculated at the prices prevailing at the beginning and end of the period) will measure the amount of nominal holding gains accruing to the owner. Exactly the same issues arise with the calculation of consumption of fixed capital and the method just described is the same as that recommended for consumption of fixed capital in the 1993 SNA (para. 12.102).

The basis for calculating depletion

Valuation of assets in the balance sheets

This section is concerned with the way in which subsoil assets should be valued in the balance sheets of the SNA. It is necessary to establish the principles governing the valuation of these assets in order to determine the way in which their sale or use should be recorded in the current and capital accounts of the System. These principles have not only to be consistent with the way in which flows and other stocks are valued in the SNA but also firmly rooted in economic theory.

The economic assets recorded in the balance sheets of the System are defined as entities (a) over which ownership rights are enforced by institutional units, individually or collectively, and (b) from which economic benefits may be derived by their owners by holding them, or using them, over a period of time.

The valuation put on economic assets represents the value of these benefits to the owner at the point in time to which the balance sheet relates. In some cases, the future benefits cannot be realised in their entirety at the moment the balance sheet is drawn up. This may be for physical reasons, that the piece of equipment is intended for use over a period of years, or because of institutional or legal arrangements as in the case of a bill or bond. In such cases, the benefits to be derived in future must be discounted to achieve their present value. This line of argument is elaborated in the SNA paragraphs 13.25 to 13.35 which read in part as follows:

“..assets and liabilities (and thus net worth) are to be valued using a set of prices that are current on the date to which the balance sheet relates and that refer to specific assets.

Ideally these prices should be observable prices on markets.

In addition to prices observed on markets, ...market prices may be approximated by the present, or discounted, value of future economic benefits expected from a given asset; this is the case for a number of financial assets, natural assets and intangible assets.

In the case of assets for which the returns are delayed (as with timber) or are spread over a lengthy period (as with sub-soil assets), although normal prices are used to value the ultimate output, a rate of discount must, in addition, be used to compute the present value of the expected future returns.”

The results of not discounting future benefits can be imagined clearly in the case of agricultural land. Supposing there is no degradation, the land has the capacity to produce the same yield year after year without limit. The sum of undiscounted future benefits is thus infinite but the fact that land changes hands for finite sums shows that the owners have a preference for money now to money in the future and discount the returns accordingly. The difference between gold in the bank and gold in the ground is that the benefits of the gold in the bank can be realised instantaneously; the benefits of the gold in the ground cannot simply because of the physical constraints on the extraction process.

Application to the valuation of natural assets

As noted above, the use of discounting to determine the present values of assets applies to natural assets just as to produced assets. The opportunity cost of depleting an asset is measured by the reduction in future production possibilities incurred by extracting and using up assets in the current period. As the cost of depleting an asset is attributable to the contraction in the production possibility frontier in the future, its measurement inevitably involves discounting. Defining the cost of depletion as the decline in the discounted value of future benefits is entirely consistent with the way in which the using up of other physical assets is recorded in the SNA. Consumption of fixed capital is defined in this way in the 1993 SNA. It is not defined in terms of allocating capital expenditures of past periods, even though it may often be estimated on the basis of such expenditures in practice. The current cost of using existing fixed assets may diverge considerably from the costs incurred in acquiring such assets in the past in the case of particularly good, or particularly bad, investment decisions.

In practice, the valuation of subsoil assets, like most other economic assets, is complicated by the fact that it depends on expectations about the future which are inevitably uncertain. Moreover, during the lifetime of an asset, the economic conditions on which such expectations are based are likely to change, causing expectations to be revised and existing assets to be revalued. The consequences of such changes will be dealt with in later sections of this paper. The first objective is to determine how subsoil assets have to be valued on the basis of a given set of economic conditions which are known to the economic agents involved and under the assumption that these will not change over the life of the asset.

The demand for a subsoil asset derives from the fact that the extracted asset - for example, oil at the well head or the coal at the pit head - may be used in the production of other goods or services. It is necessary to distinguish between the asset in the ground and the same asset after extraction at which point a market price exists. The implicit price of a subsoil asset obviously depends on both the market price of the extracted asset and the costs of extraction. The latter vary from one deposit to another depending upon geographical location, physical accessibility and the quality of the subsoil asset. These factors will influence the rate at which the asset is extracted and the costs of extraction.

The value of a subsoil asset is realised by extracting the asset and disposing of it, either by using it as an input into a further production process or selling it to another unit for the same purpose. Alternatively, the owner of a subsoil asset may sell the ownership rights over the entire deposit outright.

Thus, the benefits to the owner may be realised in three ways:

- 1) the outright sale of the entire deposit;
- 2) the progressive sale of the subsoil asset to another unit under a concession or lease whereby the lessee is licensed to extract, appropriate and dispose of the asset;

- 3) the extraction and subsequent disposal of the extracted asset by the owner, either by selling it or using it in production.

It is useful to distinguish the role of owner of the subsoil asset from that of the extractor of the asset, even when the owner carries out the extraction. This facilitates both valuation of the asset and identification of the ensuing accounting entries.

The measurement of depletion

The relationship between the current value of the sub-soil asset extracted and the value of depletion may best be explained by considering the case where the owner gradually sells off the asset to an extractor under a lease. This is conceptually helpful because it separates the returns to the fixed capital of the extractor from the returns to the owner of the asset.

The receipts from sales have to be sufficient to cover not only the cost of the depletion to the owner but provide a return on the value of the asset at the beginning of the period. The situation is exactly analogous to the renting of buildings and equipment under an operating lease where the rentals (treated as sales of services in the SNA) have to cover not only the consumption of fixed capital on the asset rented out plus any operating expenses of the lessor but also generate sufficient net operating surplus to provide a return on the value of the owner's holding of the asset.

The annex considers three scenarios based on a simple numerical example. In all cases, it is assumed that the present value of the subsoil deposit is 500 initially and the appropriate rate of discount is 10 per cent. The three scenarios cover firstly the case where there is a constant rate of extraction, secondly where the receipts from extraction remain constant over time and thirdly the case where both the extraction rate and the receipts decline exponentially. It is shown that the revenues from the sales of the asset can be partitioned into a depletion element and a residual element that represents the income accruing to the owner as a return from the ownership of the asset. The analysis and the results presented in the annex are exactly parallel to the case of determining how much operating surplus should be regarded as income and how much consumption of fixed capital in the case where only produced assets are involved.¹

The annex shows that the way in which receipts are divided between depletion and income depends on a number of factors of which the discount rate is only one. Of particular importance is the rate of extraction. The faster the resource is extracted, the greater the proportion of the receipts that must be recorded as depletion. Towards the end of the life of the deposit, this proportion increases until in the limiting case, in the year in which it is finally exhausted, all the receipts must be counted as depletion with no income component. On the other hand, if the reserves are so large that extraction does not materially affect the time over which extraction at the same rate could continue without significantly reducing the size of the reserves, then all the receipts may be regarded as income and no depletion needs to be calculated.

There is much discussion in the literature about the need to estimate the permanent income from receipts from depleting natural assets if the situation is to be sustainable. There is an implicit capital cost, or opportunity cost of holding the asset, when the owner could sell the asset for 500 and invest this sum at 10%, the assumed rate of interest, to yield a permanent income stream of 50. However, the owner does not have to sell the asset to ensure a permanent income of 50. By retaining 50 from sales receipts each year and investing the remainder at 10%, the total value of the amounts invested and accumulated interest would equal 500 at the end of the 5 years when the deposit is exhausted. In each year the interest on the

1 See for example "Profits and rates of return" by Peter Hill, OECD 1979

accumulated depletion provisions is equal to the difference between the income (receipts less depletion) and 50. This result is independent of the rate of extraction and the discount rate. The deposit can thus gradually be converted into a financial asset of equal value, if desired. This is similar to creating a sinking fund for the replacement of a fixed asset. Of course, the owner of a subsoil asset may not wish to ensure a permanent income and may decide to use the proceeds from the sale of the asset, whether outright or progressive, in other ways if preferred, just as it is possible not to create a sinking fund for the replacement of fixed capital.

Another subject of considerable discussion in the literature on valuing sub-soil assets is the so-called Hotelling case where it is assumed that the relative price of the asset rises inversely with the rate of decline of the asset so that the value of the receipts in nominal terms may remain constant indefinitely. However, in the 1993 SNA, an increase in the relative price of an asset generates real holding gains and not income and the Hotelling principle cannot be invoked either to justify life lengths assumptions or the split between income and depletion.

Entries in the flow accounts

It is now necessary to consider the implications of the extraction, sale or use of subsoil assets in the flow accounts of the SNA.

It is convenient to consider first the case when depletion and extraction are treated as separate activities undertaken by two different units, the owner or lessor on the one hand and the extractor or lessee on the other, even though both functions may sometimes be combined in a single unit.

The payments made under a concession or lease by the extractor, or lessee, to the owner, or lessor, are typically described as "royalties" or "rents"². The amounts of such payments may be determined by the physical quantities extracted or they may depend on the market value of the extracted asset. Either way they represent sales by the owner who relinquishes ownership of the subsoil asset to the lessee as the latter extracts and appropriates it. Even though the royalties or rents are often interpreted as if they were income flows, the economic reality of the situation is that the extractor is taking possession and disposing of quantities of an asset belonging to the owner. The progressive sale of an asset by the owner under a contract or lease is an alternative to the outright sale of the entire deposit to another unit. The sale of an asset is not income and the treatment of rents or royalties as primary incomes in the 1993 SNA seems conceptually inappropriate.³ The renting of land is quite different as tenants do not gradually take possession and dispose of bits of the land they are renting. At the end of a tenancy contract, the landlord expects to take back the same land in more or less the same condition as when it was rented out.

The accounts for the extractor

Ownership of the subsoil asset below ground passes from the owner to the extractor as the latter proceeds to extract and dispose of it. The production activity in which the extractor is engaged is to

2 When the owner of the asset is the government, both the government and the extractor may refer to the payments of royalties as taxes. Both SNA 1993 and ESA 1995 make clear that such payments should be treated as rents in national accounts.

3 The fact that asset sales are not income has been stressed by Salah El Serafy: see, for example, "Depletable Resources, Fixed Capital or Inventories", *Approaches to Environmental Accounting*, Alfred Franz and Carsten Stahmer (Eds.), (Physica-Verlag, Heidelberg 1993).

transform the subsoil asset into an extracted asset - for example, oil at the well head or coal at the pit head, these being the outputs of the production process in question. The proposal here is to record the subsoil asset acquired below ground as part of the intermediate consumption of this process. For example, using data for period 1 from table 2 in the annex, the value of the extracted asset is 150 so intermediate consumption of 150 would be recorded in addition to the other goods or services entering into the extractor's intermediate consumption. In consequence, the extractor's gross and net value added, and also gross and net operating surplus, are all reduced by 150 as compared with the 1993 SNA recommendations.

A consequence of regarding the payment from the extractor to the owner as a sale and intermediate consumption is to remove the entry for payment of property income. In the distribution of primary income account for the extractor, both operating surplus and property income payable are reduced by 150, leaving the balance of primary income and all subsequent accounts and balancing items unchanged.

The accounts for the owner

The owner of the asset is engaged in the activity of leasing, an activity that can be accounted for in the same way as the leasing of buildings or equipment. Initially we use a parallel with operating leasing and not financial leasing. A production account has therefore to be established for this activity, but in order to differentiate it from the extractor's production account, it will be described as a "depletion account". Using the same illustrative data for period 1 from annex table 2, a depletion account is obtained whereby resources from sales of 150 are recorded and add to gross value added. There could be additional small amounts of expenditure of an intermediate nature incurred in administering a lease, as in the leasing of buildings, but for convenience they are assumed to be negligible in the illustrative account. This means gross value added is also 150.

As already explained depletion is measured by the decline in the present value of the owner's expected receipts resulting from the extraction that occurs during the period in question. As in the case of consumption of fixed capital, depletion represents the value of an imputed transaction that takes place within an economic unit.

Depletion, like consumption of fixed capital, is a cost of production that reflects the gradual using up of the stock of some asset. As emphasised in the 1993 SNA, both value added and the operating surplus should, in principle, be measured net. The main justification for retaining gross measures within the System is the practical difficulty of estimation - a difficulty for depletion as well as consumption of fixed capital. The fact that depletion, and consumption of fixed capital, are recorded separately from intermediate consumption, thereby leading to the co-existence of both gross and net measures of value added and income, in no way diminishes the fact that they are costs of production.

In the numerical example illustrated, the owner's net value added is equal to 50, the difference between the value of the asset sold (150) and the decline in the value of the stock between the opening and closing balance sheets (100). This net value added generates a net operating surplus of 50 (assuming labour costs, taxes or subsidies are zero or negligible) which yields a 10 per cent return on the owner's capital.

The depletion account is an innovation compared with the 1993 SNA where the owner earns no operating surplus. However, matching the treatment in the accounts for the extractor, the owner no longer has property income receivable. Thus, the owner's net balance of primary income is now lower by 100 (i.e., operating surplus of 50 in place of the property income receivable of 150 previously recorded) as

compared with the 1993 SNA. Subsequent balancing items - net disposable income and net saving - are similarly reduced by 100.

However, depletion of 100 must be recorded in the owner's capital account which offsets the reduced saving of 100 leaving the owner's net lending or borrowing the same as in the 1993 SNA. Depletion, like consumption of fixed capital, is recorded as a negative change in the value of the stock of the asset. Including this entry in the capital account removes the need to record depletion in the other changes in assets account recommended in the 1993 SNA. Recording depletion in this account is not justified, given the account is reserved for changes in quantities of assets that are not attributable to economic activities and their associated transactions. By definition, depletion is an imputed transaction attributable to the activity of production and it should be recorded in the capital account as well as the production account in the same way as consumption of fixed capital and changes in inventories, both also imputed transactions.

Treating depletion in parallel with the consumption of fixed capital is entirely in keeping with the principles in the 1993 SNA of how and in which account a decrease in the value of assets due to economic use is to be recorded. It also implies that when depletion occurs, a reduction in NDP results. GDP is not affected which will disappoint some advocates of environmental accounting. However, there is no legitimate economic or accounting reason to have a valuation of product that takes account of a run down of non-produced assets but not of produced assets. The treatment proposed does, though, emphasise that for economic analysis concerned with income measures, whether or not the sustainability of these is invoked, should focus on measures of net product and not gross.

Aggregating the accounts for the extractor and the owner

Table 1 shows the effects for the extractor and owner separately and also the effect when these are aggregated. The following differences from the 1993 SNA may be noted when the accounts for the extractor and the owner are aggregated.

- 1) The reduction of 150 in the gross value added of the extractor is offset by an increase of 150 for the owner: GDP is unchanged.
- 2) The reduction of 150 in the net value added of the extractor is only partly offset by the increase of 50 for the owner: NDP is lower by 100.
- 3) The net balance of primary income and disposable income of the extractor are unchanged while those of the owner are lower by 100: NNI is lower by 100.
- 4) The capital account of the extractor is unchanged but in the capital account of the owner net saving is lower by 100 (due to lower primary income) while 100 is also recorded for depletion: net lending or borrowing for both units is unchanged. There is no impact on the financial account of either unit.

Table 1: **Entries in the accounts - Proposed changes from the 1993 SNA**

	Extractor	Owner	Combined
<i>Production account</i>			
Receipts from sales		+150	+150
Intermediate consumption	+150		+150
Gross value added - GDP	-150	+150	0
Depletion		+100	+100
Net value added -NDP	-150	+50	-100
<i>Generation of income account</i>			
Net operating surplus	-150	+50	-100
<i>Distribution of primary income account</i>			
Property income payable	-150		-150
Property income receivable		-150	-150
Balance of primary income- net - NNI	0	-100	-100
<i>Secondary distribution of income account</i>			
Disposable income - net	0	-100	-100
<i>Use of income account</i>			
Saving - net	0	-100	-100
<i>Capital account</i>			
Depletion	0	+100	+100
Net borrowing or lending	0	0	0
<i>Other changes in assets account</i>			
Depletion		-100	-100

The accounts when the owner and extractor form a single unit

When the owner and extractor are one and the same unit, no purchases and sales of the subsoil asset are recorded in the consolidated production account. Otherwise the entries for a single unit who both owns and extracts the sub-soil asset are the same as the aggregation of the accounts for separate units shown above.

While these entries might be straightforward conceptually, in practice there is considerable difficulty in determining how much of the gross operating surplus of the firm represents a return to the subsoil asset and how much a return to other assets used by the firm, particularly fixed capital. This problem is taken up elsewhere.

Identifying owners and extractors

In practice the relationships between enterprises involved in the production of oil and gas are extremely complex. Frequently a number of different enterprises are involved in some sort of consortium. There are a multiplicity of financial arrangements between the various parties. Some involve a straightforward split of costs and revenues. Others involve contingent splits and some are in effect only complex financing arrangements. These arrangements are important practically because they affect the data sources likely to be available to the national accountant. They also make clear that the paradigm used above for reasons of exposition needs further consideration.

An agreement between an extractor and an owner involves a commitment by the extractor to pay the owner an amount determined either in proportion to the value realised by the sale of the oil or fixed in absolute terms so that the payment does not move proportionally with movements in the price of oil. A more complete representation of the situation would be that the owner can put his value on the deposit only by calculating the present value of the income stream he expects from the extractor. The extractor technically possesses an intangible non-produced asset, a licence to extract the oil. The value of the licence is the present value of the excess of the returns from extracting the oil over the payments due to the owner. For economic and environmental accounting, the desired value of the deposit is the sum of these two elements. Both the extractor and owner may experience holding gains and losses on their separate non-produced assets, though in general the holding gain of the one will be the holding loss of the other.

This leads to the suggestion that it might be convenient to show the deposit entirely in the balance sheet of the extractor. A measure of depletion, calculated from the return to the deposit and initially independent of the amount contracted to the legal owner, would be shown as indicated above as a cost of production in the production account and as an entry in the capital account rather than in the other changes in assets account. The contract with the legal owner then has the character not of an operational lease (as assumed above) but of a financial lease. The initial value of the imputed financial loan is the present value at the time the agreement is entered into of the amounts to be paid by the extractor. The value is the same as the initial value of the deposit if it is assumed the owner takes sums exactly equal to the economic returns on the deposit. (As indicated, in practice the underlying value of the deposit is likely to be higher than this since the extractor also expects to make some return from it, over and above what he must pay to the legal owner.) The payments from the extractor to the owner then represent in part interest on the financial loan and also repayment of capital. Holding gains and losses will still accrue to both parties but will apply to a financial asset/liability for both the owner and extractor and will apply to a non-produced tangible asset (the deposit) for the extractor. In this case there is no need to consider the creation of the intangible non-produced asset of the licence to extract.

An alternative solution is simply to show the deposit part owned by both units. Splitting the ownership of the asset between different institutional units is exceptional within the SNA but does occur, most often when different national units combine to operate together as in the case of an airline or a central bank. When there are multiple extractors involved in a production process, it seems that some decision on splitting the assets is inevitable and including the owner in this split in no greater a complication and keeps the recording of the ownership of the asset closer to the values to be realised by each of the parties involved. Under this option also, the need to record the licence to extract as an intangible non-produced asset is also obviated.

Extensions to other environmental assets

Although the discussion above has taken oil as the example throughout, the analysis and conclusions hold for any type of sub-soil resource. Such resources are by their nature non-renewable on a human timescale. There is also a question to be answered about the need for and calculation of depletion in the case of biological assets, which are by their nature renewable, and the case of land which is marginally renewable but seldom subject to the complete absorption into other products that is the case for sub-soil assets.

Land

Although land is a non-produced asset, it has featured in the accounting conventions of the SNA for some time. The value of land for balance sheet purposes is consistent with that for fixed assets; it is valued at the present value of discounted future returns. Land under buildings does not degrade in the normal course of events so suffers neither degradation nor depletion as a result of production. All rent accruing in respect of such land is thus income. As long as land under cultivation does not suffer degradation, it too produces income with no deduction to be made in the capital account.

In fact, if agricultural land does suffer degradation, the economic rent of the land itself declines and is reflected in the production account already. The problem is that for owner-cultivated land there is no separation made between the return to the land and the return to the farmer's efforts and the decline in the rent of the land, though included in the production and generation of income accounts, is not visible. Strictly speaking, however, it can be argued that for agricultural output, value added is already measured net of degradation of land.

Land improvements

The basic treatment of land improvement in national accounts was established before the 1993 SNA. Improvements to land are treated as gross fixed capital formation. As such consumption of fixed capital is applied to the outstanding entry in the balance sheet until the original investment is written off.

The parallels with fixed capital formation are slightly forced by the need to account for a basic natural asset in what seemed an intuitively sensible way. Land improvements usually consist of construction works designed to change the character of land say from swamp to drained cultivable land, to protect against the threat of flooding or simply land clearance for building. These activities enhance the economic yield from the changed land but may not always be seen as environmental improvements. The construction works themselves do not bring economic benefits to their owners. In this case only, consumption of fixed capital is established only by writing off historic costs (adjusted to current prices) and cannot be identified as the decline in the present value of a future income stream.

The enhancement to the value of land resulting from the land improvement is shown in the other changes in assets account. The value of the enhancement need have no direct relationship with the cost of the land improvement and is not shown as a reclassification from a produced to a non-produced asset since then the diminution of the value of the land improvement would be double counted, once as consumption of fixed capital and once via a reclassification.

The alternative to treating land "improvements" as fixed capital formation would be to treat it all as current expenditure. In so far as such work is carried out by government, usually the case for major reclamation of land, there would be no effect on GDP, simply a change from capital formation to government consumption. When such work is carried out by an enterprise, treatment as current expenditure would mean increased intermediate consumption at the start of a project and lower consumption of fixed capital subsequently. This evokes comparison with the decision in the 1993 SNA to treat mineral exploration as fixed capital. Here too the activity of exploration is linked to another non-produced asset (a sub-soil resource) expected to bring benefits over a long time scale. For both land improvement and mineral exploration, the SNA nods to commercial accounting principles (and thus data sources) where such "lumpy" expenditures may be spread over a number of years by initial treatment as investment and subsequent write off.

Biological resources

These may be either cultivated or non-cultivated. Cultivated resources are those subject to human control and include cultivated forests, organised crops and animal stocks. Cultivated resources are all included within the SNA asset boundary as produced assets. In relatively small and densely populated countries all or almost all biological resources may in fact fall in this category.

Cultivated resources

Consider the cases of cultivated forests, fish farms or livestock herds. Those yielding repeat products are classified as fixed assets; those yielding once-only products are classified as inventories. For cultivated resources classed as inventories, the closing stocks at the end of the year are equal to the opening stocks plus natural growth less offtake and any other changes in volume (catastrophic losses, changes in classification etc.) all adjusted for price changes. The value of output is equal to natural growth; the value of the offtake is equal to sales, which may be intermediate consumption or exports, and the excess or deficit of natural growth over offtake is recorded as changes in inventories.

For cultivated resources classified as fixed capital, natural growth represents output and capital formation. The yield of repeat products is also output and sales. The effects of ageing and death of livestock should in principle be recorded as consumption of fixed capital. This should cover both death due to natural causes and also a deliberate run-down of stock, for example in response to bad weather or other considerations of good management since the expected future income from the repeat products of the animals suffers a corresponding decline.

Data sources may not permit such a clear recording in the accounts as this. Some livestock herds are used both for milk or wool and meat and may be classified as capital formation despite the ultimate single use. Further, natural growth may often only be measured net by comparing populations from one year to the next. In such circumstances, natural growth treated as capital formation will in fact be a measure of net and not gross capital formation and may exceptionally be negative. In this case, as in the

case of degrading agricultural land, the SNA already deducts from GDP the effects of run-down of environmental assets.

Non-cultivated biological resources

One difference between cultivated and non-cultivated biological resources is that the coverage of the latter may change as a result of new discoveries, new forms of access or new technologies (e.g. the advent of freezer ships). Such changes in stocks should clearly be accounted for in the other volume changes in assets account.

If offtake exceeds natural growth, there is a case to be made to say the excess should be treated as depletion as in the case of sub-soil assets.

The question arises of who “owns” the assets to know in which capital account to record the depletion. For commercial logging, for example, there is an identifiable “extractor” as in the mining case who takes control, if not legal ownership, of the asset. When the depletion is due to subsistence or other very small scale activity which may well happen in developing countries but may also occur in the case of fishing, it is less clear that it makes sense to show these (household) units as suffering depletion in a capital account. Since these assets are owned collectively, it may be preferable to show the depletion loss as occurring in a notional unit separate from other institutional units and sectors in the System. This notional unit would represent “the environment”. An alternative would be to ascribe the ownership to general government but it would seem appropriate in that case to keep this aspect as a separate sub-sector of government to avoid consolidation of these collectively owned assets from others purchased by government.

If the offtake of non-cultivated biological resources is less than natural growth, the situation is sustainable and the whole value of sales may be counted as income. This is the default assumption in both the 1968 and 1993 SNA. Should any excess of natural growth over offtake should be treated as output? At first sight, this is simply the symmetric alternative to the case in the previous sentence and the answer seems to be yes. However, there is a further aspect to be considered. If the offtake is controlled by legislation, is this control sufficient to constitute “direct control, responsibility and management of institutional units”? If so, the resources become fixed assets and not non-produced assets and the treatment becomes the same as described above for cultivated resources. If not, but it is still thought appropriate to include an excess of natural growth over offtake as production, then the case for distinguishing cultivated and non-cultivated assets seems to break down since both will receive the same treatment.

The proposal here, therefore, is that while any depletion of non-cultivated assets should appear in the capital account and reduce NDP, any net increase in these assets should remain in the other volume changes account and not augment GDP.

Natural resource depletion and discoveries

The discussion so far has presumed that depletion occurs and the problem is how to account for it in the System. There is in fact a prior question of determining when depletion occurs.

In calculating depletion for natural resources, it is necessary to have an estimate for the life length of the resource. This is usually calculated by dividing the stock at the beginning of the year by the extraction in the year. When the extent of a resource is known exhaustively and there are no possibilities of new discoveries (as may be the case for a phosphate atoll or a tropical rainforest) this seems wholly

appropriate. It is more questionable in the case of petroleum reserves. Both the SNA and the commercial accounting standards quote only “proven” reserves on the balance sheet. However, these augment regularly by the advent of new discoveries and new technologies which convert previously known but uneconomic fields from an unproven to proven status. Over the long term, oil companies aim to “prove” reserves about as quickly as they deplete existing proven reserves so the level shown in the balance sheet stays more or less constant.

In existing examples of calculating depletion of oil reserves, for example the recent UK article in *Economic Trends*⁴, the discoveries are added to the stocks at the beginning of the next year and affect the calculation of life length in year $t+1$ but do not affect the calculations for year t . This keeps the life length of the reserve fairly short and the value of depletion significant but is in keeping with the proposition in the SNA that the value of new discoveries should not count as income but be shown in the other changes in assets account.

An alternative is proposed here to calculate life length differently, as the stock at the beginning of the year divided by the decrease between the start of year and end of year stocks. This is in keeping with the idea of sustainability; if there is no reduction in the stock levels there is no need to allow for depletion. It is consistent with the treatment of renewable natural assets where the level of depletion is calculated as offtake less natural growth; here discoveries are the counterpart of natural growth. The effect of this is to extend the life length in years where discoveries are less than extraction and to remove the need for depletion in years where discoveries exceed extraction. (Because of the lumpiness of the process, it may be more realistic to use a moving average of, say, three or five years.)

What is the implication of this for the measurement of income? While accepting the proposition that it is inappropriate to count the whole value of the discovery as income in the same year, it can be argued that there is a long term income effect from it. This may be likened to the annuity value of a windfall gain from a lottery. In the case under consideration, the value of the income effect in the year of the discovery is the reduction in the depletion allowance by the effect of extending the life length of the reserve. As pointed out above, in fact this income effect is taken into account from year $t+1$ onwards so allowing it to affect year t also is perhaps less innovative than it seems at first sight.

Simple as the adjustment to the calculation is, the effects on the calculations could be quite significant as illustrated in the example in table 2. Suppose a constant depletion of 10 from an initial value of 100. Over an eight year period, extraction of 80 is offset by discoveries of 60, a run-down overall of 20 per cent. Taking extraction over start of year stocks only, the average life length decreases from 10 years to 7. If three year averages of the rate of discoveries is used, the average life length starts at 45 and declines but only to 28 years. Clearly the allowance for depletion would be very significantly less under the second scenario. In this example, there is still a significant decline in stocks between the beginning and end of the period. Based on experience to date of the pattern of discoveries and extraction of oil and gas, it is likely that little if any depletion would be appropriate in respect of these deposits under this proposal. This would make a distinction between sub-soil resources known to exhaustion and those of indeterminate extent but a distinction consistent with the concept of sustainability.

4 Reference

Table 2: **Effect of discoveries on life length**

Year	1	2	3	4	5	6	7	8
Start of year stock	100	90	98	94	84	89	85	75
Extraction in year	10	10	10	10	10	10	10	10
Stocks at end of year, after extraction, before discoveries	90	80	88	84	74	79	75	65
Life length based on extraction only	10	9	9.8	9.4	8.4	8.9	8.5	7.5
Discoveries		18	6		15	6		15
End of year stock	90	98	94	84	89	85	75	80
Average discoveries (centered three year)	*	8	8	7	7	7	7	*
Life length based on extraction and discoveries		45	49	31	28	30	28	

If discoveries are to be treated in this way, consideration needs to be given to whether other changes in the volume of assets should also be regarded as having an income effect via life expectancies. It seems realistic for catastrophic losses and changes to proven reserves because of changes in technology. On the other hand changes in proven reserves because changes in relative prices affect the economic viability of known reserves are holding gains and losses and do not result in an associated income effect.

Concluding Remarks

The analysis of the causes of changes in the values of assets between the opening and closing balance sheets leads to the identification of appropriate entries in the various flow accounts of the System. For produced assets, and even for improvements to land, all changes resulting from economic transactions are entered in accounts between the production and financial account inclusive. Only non-transactional changes are recorded in the other volume changes in assets account and the revaluation account. The case argued here is that the rules of the System, without modification, suggest that depletion and degradation of non-produced assets that are the results of economic activities should also be excluded from the Other Volume Changes in Assets and Revaluation accounts and find their place in the other flow accounts by adopting similar valuation and classification principles as for produced assets. To underline the parallel proposed between economic and natural assets, table 3 shows where different causes of changes to the value of assets are (or it is proposed should be) classified in the different accounts.

The treatment proposed stems directly from applying the accounting principles of the System to all assets within the System's asset boundary. In marked contrast, though, the effects of changes in the

quality of natural assets that are not incorporated within the System's asset boundary (degradation of air and water by their use as environmental sinks) do not lend themselves immediately to the same treatment. To attempt to do so either (i) dislocates the accounting links between the flow and stock accounts, or (ii) introduces notional prices and by implication notional effects which would affect many entries in the accounts, including the net present value calculations of economic assets. Neither flow nor stock effects can be calculated directly in their entirety and it is proposed that such calculations belong in an ancillary system, essentially of a modelling rather than an accounting nature, which it is not appropriate at this stage to think of incorporating in the SNA itself.

The practical problems of measuring depletion in a theoretically satisfactory way may be formidable. For this reason, it may take some time before the kind of measures proposed in this paper can be implemented. The situation is similar to that already existing for consumption of fixed capital which many countries still find difficult to estimate satisfactorily. For this reason, the SNA still has to live with gross measures of value added and income, notwithstanding the theoretical superiority of net measures. However, recognition of the validity in principle of incorporating depletion within the flow accounts of the System would be a major step towards meeting the demand for integrating environmental considerations within the System, a demand recognised as a desirable development in the 1993 SNA itself.

STD/NA/RD(97)7
Table 3: Entries in the system relating to various assets

	Produced asset, e.g. machinery	Produced asset - inventories	Non-produced asset - sub-soil deposits	Produced asset - mineral exploration	Non-produced asset - land	Produced asset - land improvement	Produced asset e.g. cultivated livestock	Non-produced asset e.g. non-cultivated livestock
Opening balance sheet	Market prices =NPV	Market prices	NPV	Nil activity before	Market prices =NPV	Nil activity before	Market prices	Market prices
Production/depletion account	Gross benefits included in gross operating surplus	Changes in inventories	Benefits included in gross operating surplus	Value of output undertaken	Rent (may be offset in part by effects of degradation)	Value of output undertaken	Natural growth (probably net of natural wastage)	Offtake
Capital account	Consumption of fixed capital (CFC)	Changes in inventories	Depletion of natural assets	Gross capital formation (= value of output) less CFC		Gross capital formation (= value of output) less CFC	Output offtake is gross capital formation or changes in inventories depending on nature of livestock	Offtake natural growth(if +ve) shown as depletion of natural assets
Other volume change in assets account	Changes in technology, change in asset lives, catastrophic losses	Catastrophic losses	New discoveries, effect of new technologies	Associated with but not equivalent to new discoveries	Catastrophic losses	Associated with but not equivalent to increases in land value	Changes in classification, catastrophic losses	Natural growth less offtake (if +ve), new discoveries etc, catastrophic losses
Revaluation account	Price changes	Price changes	Change in interest rates	Price changes	Price changes (including those arising from land improvement)	Price changes	Price changes	Price changes
Closing balance sheet	Market prices =NPV	Market prices	NPV	Written down value of output	Market prices (=NPV)	Written down value of output	Market prices	Market price

ANNEX: EXAMPLES OF THE CALCULATION OF DEPLETION ALLOWANCES

This annex begins by examining three common scenarios for the calculation of depletion for a subsoil deposit. In each case it is assumed the present value of the expected receipts before extraction starts is 500 and that the appropriate rate of discount is 10 percent. The three cases to be examined are:

- 1) a situation in which the rate of extraction, and consequential flow of receipts to the owner, is constant from period to period;
- 2) a situation in which the present value of the owner's expected receipts declines from period to period at a constant linear rate;
- 3) a situation in which the rate of extraction and consequential flow of receipts declines by a constant percentage rate from period to period.

All three cases have attracted attention in the literature on "economic depreciation".

(1) Constant rate of extraction

The first case is illustrated in table 1. The extraction process is assumed to take place at a constant rate and to take 5 periods to complete. This implies that the value of the receipts must be 131.9 per period. The first case is therefore equivalent to a 5 period lease yielding 131.9 each period with a discount rate of 10 per cent. Receipts will be denoted by f_t , the discount rate by r and the present value of the remaining receipts at the start of each period by v_t .

500 is also the amount that the owner could expect to realise from outright sale of the entire deposit to another unit. A potential buyer would be unlikely to pay more while the owner would be unlikely to accept less, so that there is no conflict, in principle, between valuation at the current market price obtainable for the sale of the entire deposit and valuation on the basis of the present value of the future receipts from the progressive sale of the subsoil asset under a leasing contract.

As the process of extraction occurs, the value of the deposit to the owner gradually diminishes. The present value of the receipts from the sales of the remaining assets at the start of period t is given by

$$v = \sum_{i=t}^n \frac{f_i}{(1+r)^{i-t+1}} \tag{1}$$

This amount is shown in the second column of the table. The decline in the present value of the remaining subsoil assets between the opening and closing sheets in year t is given by

$$\begin{aligned} d_t &= v_t - v_{t+1} \\ &= f_t - rv_t \end{aligned} \quad (2 \text{ and } 3)$$

Equation (3) is valid in general whether or not the f's are constant. When the decline in the balance sheet values is entirely attributable to the process of extraction it must be the appropriate measure of depletion.

Defining depletion in terms of the decline in the present value of the future receipts is the generally accepted measure of the depreciation of an asset in economics. It was originally proposed by Hotelling in 1925 and its properties are well established⁵. Consumption of fixed capital is defined this way in the 1993 SNA (see paras. 6.179 to 6.200) and the measure of depletion proposed here is entirely consistent with the way in which assets in general are accounted for in the 1993 SNA.

Table 1: **Constant Receipts**

Period	Receipts from sale of subsoil asset in period t	Present value of remaining future receipts at the start of period t	Value of depletion in period t	Receipts minus depletion in period t
	(f_t)	(v_t)	(d_t)	($f_t - d_t$)
1	131.9	500.0	81.9	50.0
2	131.9	418.1	90.1	41.8
3	131.9	328.0	99.1	32.8
4	131.9	228.9	109.0	22.9
5	131.9	119.9	119.9	12.0
Total	659.5		500.0	159.5

In the numerical example, illustrated in part I of the table, the values of depletion as defined in equations (2) and (3) above are given in the third column. For example, depletion in year 1 is given by (500 - 418.1), the difference between the opening and closing balance sheet values for year 1. Alternatively, it can be calculated from equation (3) as

5 See H. Hotelling: "A General Mathematical Theory of Depreciation", Journal of the American Statistical Association, September 1925, pp. 340-353. For a discussion of the properties of economic depreciation see, for example, J.E. Meade, A Neo-Classical Theory of Economic Growth, Appendix 3; T.P. Hill, Profits and Rates of Return, OECD, Paris, 1979 Chapter 2; D.W. Jorgensen: "Capital as a Factor of Production" in Technology and Capital Formation, D.W. Jorgensen and R. Landau, (Eds.) Cambridge, MIT Press, 1989, pp. 1-35.

$$d_t = 131.9 - 0.1 \times 500 = 81.9$$

It can be seen that the value of depletion is less than the value of the subsoil asset extracted and sold each period and tends to increase over time in this example. Rearranging (3) we have

$$f_t = d_t + rv_t \tag{4}$$

(2) *Constant rate of depletion*

If the value of depletion, d_t , is to be constant from period to period, the sequence of f_t 's has to decline from period to period at the same rate as rv_t in expression (3). Setting d_t equal to v_t/n where v_t is the present value of the asset before extraction commences, it follows from (3) that:

$$f_t = rv_t + \frac{v_t}{n} \tag{5}$$

The f_t 's form a declining arithmetic progression. As in the previous example, choosing $v_t = 500$, $n = 5$ and $r = 0.1$ the sequence of f_t 's shown in part 2 of the table are obtained. The f_t 's decline by 10 each period from 150 in period 1 to 110 in period 5 while depletion remains constant at 100 per period. This case is, of course, equivalent to straight-line depreciation in the case of consumption of fixed capital.

As before, the owner may, if he wishes, convert the proceeds from the progressive sale of the asset into a permanent income stream of 50 (i.e., 10 per cent of v_t , the initial value of the asset) by retaining 50 each period out of the receipts from sales and investing the remainder at 10 per cent. The total of the amounts invested plus cumulated interest is equal to 500 at the end of the lease.

Table 2: **Constant depletion**

Period	Receipts from sale of subsoil asset in period t (f_t)	Present value of remaining future receipts at the start of period t (v_t)	Value of depletion in period t (d_t)	Receipts minus depletion in period t ($f_t - d_t$)
1	150	500	100	50
2	140	400	100	40
3	130	300	100	30
4	120	200	100	20
5	110	100	100	10
Total	650		500	150

(3) Geometrically declining rate of extraction

Assume that the rate of extraction, and hence the value of receipts from the sale of the subsoil asset, f_t , decline indefinitely at the constant rate δ per period. That is,

$$\text{assume } f_t = (1-\delta) f_{t-1}, \text{ where } 0 < \delta < 1 \quad (6)$$

Given that, in general, with economic depreciation

$$f_t = d_t + rv_t, \quad (7)$$

it follows that

$$v_t = (1-\delta)v_{t-1} = v_1(1-\delta)^{t-1} \quad (8)$$

$$d_t = v_t - v_{t+1} = \delta \cdot v_1(1-\delta)^{t-1} = \delta \cdot v_t \quad (9)$$

$$f_t = \delta \cdot v_t + rv_t = (\delta + r)v_t \quad (10)$$

The numerical example in the third part of the table is constructed assuming that v_t , the present value of the flow of receipts at the start of period 1, is 500 as before, and that the rate of decline in the receipts is 20 per cent per period and the discount rate is 10 per cent. Substituting these parameters into the above equations numbers in the table are obtained.

In this example, the value of depletion is equal to two-thirds of the value of the receipts of each period. It can be seen from (9) and (10) above that, in general, the ratio of d_t to f_t is constant: thus,

$$\frac{d_t}{f_t} = \frac{\delta}{(\delta + r)} \quad (11)$$

The faster the rate of decline in the receipts relatively to the discount rate, the larger the proportion of the receipts attributable to depletion. It may be noted that the ratio of depletion to the present value of the stock of the asset at the start of each period, namely d_t/v_t , is also constant and equal to δ .

If the rate of decline is equal to the discount rate it follows that depletion is equal to one half of the value of the asset extracted each period. This special case has attracted some attention in the theoretical literature on economic depreciation because of its mathematical convenience.

It is already clear from the preceding two cases, that the owner of the asset ought to be able to obtain a permanent income of 50 (i.e. rv_t ,) out of the proceeds from the sale of the subsoil asset in this case also by investing at the going rate of interest of 10 per cent the amount by which receipts from sales and interest exceed 50 in each period. The value of the declining sales receipts is exactly offset by increasing interest receipts to ensure a steady income of 50. However, the previous examples show that any pattern of sales (including outright sale of the entire deposit) can provide an income of 50 and it is not necessary to assume that extraction declines exponentially to ensure a steady income.

Table 3: Geometrically declining receipts and depletion

Period	Receipts from sale of subsoil asset in period t	Present value of remaining future receipts at the start of period t	Value of depletion in period t	Receipts minus depletion in period t
	(f_t)	(v_t)	(d_t)	$(f_t - d_t)$
1	150	500	100	50
2	120	400	80	40
3	96	320	64	32
4	76.8	256	51.2	25.6
5	61.4	204.8	40.9	20.5
--	--	--	--	--
Total	750		500	250

Relationship to the "user-cost" method

There are some points of similarity between the method proposed in this paper and the "user-cost" method proposed by Salah El Serafy. It is useful therefore to clarify exactly what they have in common and how they differ. Both certainly agree on one fundamental point of principle, namely that the sale or disposal of an asset is not an income flow. Nevertheless, El Serafy seeks to identify how much of the value of the sales or disposals of a subsoil asset can be treated as income, the remainder being retained for investment in an income bearing asset to ensure that the income can be maintained indefinitely. The latter component is described as the "user-cost" or depletion factor.

The present paper defines depletion as the value of the economic depreciation on the stock of the subsoil asset in the tradition of Hotelling. When the extraction rate and associated flow of receipts are constant from period to period and the stock of the asset is finite and exhaustible (illustrated in part I of the table) the ratio of depletion to receipts in the first period is given by:

$$\frac{d_1}{f_1} = \frac{1}{(1+r)^n} \tag{15}$$

This follows from the basic definitions in equations (1) to (3) and the assumption that f, is constant. For example, in period I of part I of the table:

$$\frac{81.9}{131.9} = \frac{1}{(1+0.1)^5} \tag{16}$$

Equation (15) is the same as El Serafy's formula for the calculation of the share of depletion in the value of receipts.

In the present paper the difference between f_t and d_t , e.g., between 131.9 and 81.9, is identified with the owner's net value added and net operating surplus. It follows from the basic definitions that it must equal the owner's return on the value of the asset at the start of the period, rv_t . As the depletion proceeds, this value added and operating surplus gradually declines, as indeed the return on the shrinking stock of an asset must. As noted in this paper, however, this decline can be exactly compensated by increased interest received from the investment of the depletion provisions, provided the owner chooses to invest the whole of the depletion charges. In this way, although the owner's operating surplus declines, the owner's net primary income (consisting of operating surplus plus interest received) can be sustained indefinitely, if desired. This is, of course, equivalent to El Serafy's proposal. However, in periods after the first, it is not possible to define depletion as the difference between receipts and this hypothetical constant primary income.

In general, when the receipts are constant the ratio of depletion to receipts is given by:

$$\frac{d_t}{f_t} = \frac{1}{(1+r)^{t+1}} \quad (17)$$

This reduces to El Serafy's formula only when $t=1$. If depletion provisions are calculated in every period according to El Serafy's formula, their cumulative value is insufficient to cover the initial value of the stock of the asset.

Thus, according to the methodology proposed here, even when the flow of receipts is constant El Serafy's formula is valid only in the first period. When the rate of extraction and flow of receipts are not constant, the results obtained here are quite different from those given by El Serafy's formula. For example, as n tends to unity, El Serafy's formula tends to zero. Indeed, if a constant rate of extraction can be maintained in perpetuity the supply of the asset is, in effect, inexhaustible so that the value of depletion does become zero. This case is not very relevant therefore. However, when the total stock of the asset is limited and exhaustible and the rate of extraction declines geometrically, the limiting case in which n tends to infinity does become highly pertinent. In this case, the ratio of depletion to receipts is constant: as shown above,

$$\frac{d_t}{f_t} = \frac{\delta}{\delta + r} \quad (18)$$

This formula may provide a useful approximation to depletion in many situations where the rate of extraction is in fact gradually diminishing over time.