Chart 1. ECP and SNA real GDP growth rates (1996-2000)

Chart 1. ECP and SNA real GDP growth rates (1996-2000)
Why are the growth rates different?

- Putting two countries and time periods together
  - Implicit Deflator (temporal)
    \[
    \sum_i w_{it} \frac{p_{it+k}^S}{p_{it}^S} \quad \text{and} \quad \sum_i w_{it} \frac{p_{it+k}^R}{p_{it}^R}
    \]
  - PPP (spatial)
    \[
    \left( \sum_i w_{it} \frac{p_{it}^S}{p_{it}^R} \right)^{1/2} \quad \text{and} \quad \left( \sum_i w_{it} \frac{p_{it}^S}{p_{it}^R} \right)^{1/2}
    \]
    \[
    \left( \sum_i w_{it} \frac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2} \quad \text{and} \quad \left( \sum_i w_{it} \frac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2}
    \]
  - Reconciliation must be spatial-temporal
    ✓ Cross country comparison of growth rates
Building the answer

PPP implied growth in $S$

$$ECP^S_t = \frac{\sum_i p_{it}^R q_{it}^R}{\sum_i p_{it} q_{it}} \left[ \frac{\sum_i p_{it+k}^S q_{it+k}^S}{\sum_i p_{it+k}^R q_{it+k}^R} \right]^{1/2} \left( \frac{\sum_i w_{it+k}^S p_{it+k}^S}{\sum_i w_{it+k}^R p_{it+k}^R} \right)^{1/2} + \varepsilon_{t+k}$$
Building the answer

Multiplying and dividing by the real growth in $S$ and rearranging terms

$$ECP_t^S = \frac{\sum_i p_{it+k}^S q_{it+k}^S}{\sum_i p_{it+k}^S q_{it+k}^S} \cdot \frac{\sum_i p_{it+k}^S}{\sum_i p_{it+k}^R} \left[ \left( \frac{\sum_i w_{it+k}^S p_{it+k}^S}{\sum_i w_{it+k}^S p_{it+k}^R} \right)^{1/2} \left( \frac{\sum_i w_{it+k}^R p_{it+k}^S}{\sum_i w_{it+k}^R p_{it+k}^R} \right)^{1/2} + \varepsilon_{t+k}^{SR} \right]^{-1}$$
Building the answer

- The ratio of implicit deflators as a function of component level SNA-based PPPs

\[
\frac{\sum_i p_{it+k}^S q_{it+k}^S}{\sum_i p_{it+k}^R q_{it+k}^R} = \frac{\sum_i p_{it+k}^R}{\sum_i p_{it+k}^R} \frac{\sum_i p_{it+k}^S q_{it+k}^S}{\sum_i p_{it+k}^S q_{it+k}^R} = \frac{\sum_i w_{it+k}^R q_{it+k}^R}{\sum_i w_{it+k}^R} \frac{S}{R(t)} \frac{p_{it+k}^R q_{it+k}^R}{p_{it+k}^R q_{it+k}^R}
\]

- Weights observable but quantity ratios not
  - need quantity ratios to operationalize
Finding the Quantity Ratios

From the National Accounts we have:

\[ p_{it}^{S} q_{it}^{S} = V_{it}^{S} \quad (1) \Rightarrow \quad \frac{p_{it}^{S} q_{it}^{S}}{p_{it}^{R} q_{it}^{R}} = \frac{V_{it}^{S}}{V_{it}^{R}} \quad (7) \]

\[ p_{it}^{R} q_{it}^{R} = V_{it}^{R} \quad (2) \]

\[ p_{it}^{S} q_{it}^{S} = V_{it}^{S} \quad (3) \Rightarrow \quad \frac{p_{it}^{S} q_{it}^{S}}{p_{it}^{R} q_{it}^{R}} = \frac{V_{it}^{S}}{V_{it}^{R}} \quad (8) \]

\[ p_{it}^{R} q_{it}^{R} = V_{it}^{R} \quad (4) \]

\[ \frac{p_{it}^{S} q_{it}^{S}}{p_{it}^{R} q_{it}^{R}} = \frac{V_{it}^{S}}{V_{it}^{R}} \quad (5) \Rightarrow \quad \frac{p_{it}^{S} q_{it}^{S}}{p_{it}^{R} q_{it}^{R}} = \frac{V_{it}^{S}}{V_{it}^{R}} \quad (9) \]
Finding the Quantity Ratios

The National Accounts Equations:

\[
\frac{S}{p_{it} R_{it}} = \frac{V_{it}^S}{V_{it}^R} \quad (7)
\]

\[
\frac{S}{p_{it + k} R_{it + k}} = \frac{V_{it + k}^S}{V_{it + k}^R} \quad (8)
\]

\[
\frac{S}{p_{it + k} R_{it + k}} = \frac{S(t)}{K_{it + k}^S(t)} \quad (9)
\]

Are supplemented by ECP-based level information

\[
\frac{p_{it}^S}{p_{it}^R} = \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \quad (10)
\]
Finding the Quantity Ratios

Solving gives:

\[
\frac{p^i_t}{q^i_t} \cdot \frac{S}{R} = \frac{V^i_S}{V^i_R} \quad (7) \quad \text{and} \quad (10) \quad \Rightarrow \quad \frac{q^i_t}{q^i_t} \end{equation} = \frac{R}{R} \Rightarrow \frac{V^i_S}{V^i_R} \quad (11)
\]

\[
\frac{p^i_t}{q^i_t + k} = \frac{S}{q^i_t + k} \quad \Rightarrow \quad \frac{R}{R} \Rightarrow \frac{V^i_S}{V^i_R} \quad (12)
\]

\[
\frac{S}{R} = \frac{V^i_S}{V^i_R} \quad (8) \quad \text{and} \quad (12) \quad \Rightarrow \quad \frac{q^i_t + k}{q^i_t + k} \end{equation} = \frac{R}{R} \Rightarrow \frac{V^i_S}{V^i_R} \quad (13)
\]

\[
\frac{p^i_t}{p^i_t + k} = \frac{S}{p^i_t + k} \quad (10) \quad \text{note} \quad \frac{R}{R} \Rightarrow \frac{V^i_S}{V^i_R} \quad (14)
\]
Implications for Decomposition of real GDP growth rates

We can now rewrite the equation in slide 4 as:

\[
ECP_t^S = \frac{\sum_i p_{it+k}^S q_{it+k}}{\sum_i p_{it+k}^S} \frac{\sum_i W^R_{it+k} R_{it+k}}{p_{it+k}^S} \left[ \left( \sum_i \tilde{w}_{it+k}^S \frac{\tilde{p}_{it+k}^S}{R_{it+k}} \right)^{1/2} \left( \sum_i \tilde{w}_{it+k}^S \frac{\tilde{p}_{it+k}^S}{R_{it+k}} \right) \right]^{-1} + \varepsilon_{SR}^{SR}
\]

where

\[
W^R_{it+k} = \frac{R_{it+k}}{p_{it+k}^S} \frac{R(t)}{p_{it+k}^S} \frac{K_{it+k}^S(t)}{K_{it+k}^S(t)} \quad \text{and} \quad W^R(t)_{it+k} = \frac{R(t)}{p_{it+k}^S} \frac{R(t)}{p_{it+k}^S} \frac{K_{it+k}^S(t)}{K_{it+k}^S(t)}
\]

- Progressive substitution of ECP-based prices, then weights, in the middle term causes it to converge towards the right term. Each substitution generates an implied SNA growth rate that can be used to summarise the impact of that substitution.
Implications for Decomposition of real GDP growth rates

- After all price substitutions we have:

$$ECP_t^S = \frac{\sum_i p_{it+k} q_{it+k}}{\sum_i p_{it+k} q_{it}} \cdot \frac{\sum_i W_{it+k} R_{it+k}}{\sum_i W_{it+k}} \left( \frac{\sum_i \tilde{w}_{it+k} p_{it+k}}{\sum_i \tilde{w}_{it+k}} \right)^{1/2} \left( \frac{\sum_i \tilde{w}_{it+k} p_{it+k}}{\sum_i \tilde{w}_{it+k}} \right)^{1/2} + \varepsilon_{SR}$$

- The right two terms can be written:

$$\frac{\sum_i W_{it+k} R_{it+k}}{\sum_i W_{it+k}} \left( \frac{\sum_i \tilde{w}_{it+k} p_{it+k}}{\sum_i \tilde{w}_{it+k}} \right)^{1/2} \left( \frac{\sum_i \tilde{w}_{it+k} p_{it+k}}{\sum_i \tilde{w}_{it+k}} \right)^{1/2} + \varepsilon_{SR}$$
Implications for Decomposition of real GDP growth rates

Substituting ECP-style weights using current SNA:

\[
\left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \left[ \left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \right]^{-1} + \varepsilon_{SR} + \varepsilon_{t+k}
\]

- gives the effect of weighting formulas

Substituting original ECP weights

\[
\left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \left[ \left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \left( \Sigma_i \tilde{w}_{it+k} \frac{\tilde{p}_{it+k}}{\hat{p}_{it+k}} \right)^{1/2} \right]^{-1} + \varepsilon_{SR} + \varepsilon_{t+k}
\]

- gives the effect of weight revisions
Ø

EKS effect is given by deviation from 1 of:

\[
\left( \sum_i \tilde{w}_i^S \frac{\tilde{p}_i^{it+k}}{\tilde{p}_i^{it+k}} \right)^{\frac{1}{2}} \left( \sum_i \tilde{w}_i^R \frac{\tilde{p}_i^{it+k}}{\tilde{p}_i^{it+k}} \right)^{\frac{1}{2}} \left[ \left( \sum_i \tilde{w}_i^S \frac{\tilde{p}_i^{it+k}}{\tilde{p}_i^{it+k}} \right)^{\frac{1}{2}} \left( \sum_i \tilde{w}_i^R \frac{\tilde{p}_i^{it+k}}{\tilde{p}_i^{it+k}} \right)^{\frac{1}{2}} + \varepsilon_i^{SR} \right]^{-1}
\]
Data Considerations:

- Expenditure Categories
  - Actual individual consumption
  - Collective consumption of general government
  - Gross Fixed Capital Formation
  - Changes in inventories & acquisitions less disposals of valuables
  - Exports of goods & services (SNA)
  - Imports of goods & services (SNA)
  - Trade balance (ECP)
  - Gross Domestic Product
### Table 1. Cumulative Effect of Price and Weight substitutions on 1996-2000 SNA growth rates

<table>
<thead>
<tr>
<th>Country</th>
<th>ECP</th>
<th>PPP1</th>
<th>weight2</th>
<th>weight1</th>
<th>PPP2</th>
<th>SNA</th>
<th>Linking/EKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belarus</td>
<td>-22.8</td>
<td>-14.7</td>
<td>-17.6</td>
<td>21.9</td>
<td>-17.6</td>
<td>41.1</td>
<td>-5.2</td>
</tr>
<tr>
<td>Kyrgyzstan</td>
<td>-30.4</td>
<td>-17.9</td>
<td>-33.5</td>
<td>-7.9</td>
<td>-33.5</td>
<td>19.7</td>
<td>3.1</td>
</tr>
<tr>
<td>Armenia</td>
<td>-6.9</td>
<td>3.9</td>
<td>-14.8</td>
<td>-2.6</td>
<td>-14.8</td>
<td>24.5</td>
<td>7.9</td>
</tr>
<tr>
<td>Republic of Moldova</td>
<td>-34.2</td>
<td>-17.5</td>
<td>-26.9</td>
<td>-16.9</td>
<td>-26.9</td>
<td>-6.1</td>
<td>-7.3</td>
</tr>
<tr>
<td>The FYR of Macedonia</td>
<td>44.4</td>
<td>83.0</td>
<td>42.9</td>
<td>-2.2</td>
<td>42.9</td>
<td>18.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Iceland</td>
<td>2.5</td>
<td>9.0</td>
<td>10.7</td>
<td>-31.5</td>
<td>10.7</td>
<td>22.7</td>
<td>-8.2</td>
</tr>
<tr>
<td>Kazakhstan</td>
<td>-15.0</td>
<td>-2.8</td>
<td>-5.7</td>
<td>1.4</td>
<td>-5.7</td>
<td>4.1</td>
<td>-9.3</td>
</tr>
<tr>
<td>Lithuania</td>
<td>28.9</td>
<td>35.4</td>
<td>35.2</td>
<td>14.0</td>
<td>35.2</td>
<td>16.4</td>
<td>-6.3</td>
</tr>
<tr>
<td>Croatia</td>
<td>24.0</td>
<td>23.4</td>
<td>18.6</td>
<td>2.4</td>
<td>18.6</td>
<td>11.8</td>
<td>5.4</td>
</tr>
<tr>
<td>Slovenia</td>
<td>11.6</td>
<td>15.5</td>
<td>18.7</td>
<td>11.5</td>
<td>18.7</td>
<td>19.3</td>
<td>-7.1</td>
</tr>
<tr>
<td>Ireland</td>
<td>40.5</td>
<td>36.0</td>
<td>52.2</td>
<td>44.3</td>
<td>52.2</td>
<td>47.4</td>
<td>-11.7</td>
</tr>
<tr>
<td>Finland</td>
<td>18.9</td>
<td>21.5</td>
<td>29.5</td>
<td>18.2</td>
<td>29.5</td>
<td>24.5</td>
<td>-10.7</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>10.4</td>
<td>21.2</td>
<td>22.0</td>
<td>6.3</td>
<td>22.0</td>
<td>5.3</td>
<td>-11.7</td>
</tr>
<tr>
<td>Sweden</td>
<td>11.7</td>
<td>15.4</td>
<td>20.1</td>
<td>8.4</td>
<td>20.1</td>
<td>16.0</td>
<td>-8.4</td>
</tr>
<tr>
<td>Poland</td>
<td>17.4</td>
<td>22.3</td>
<td>19.8</td>
<td>10.6</td>
<td>19.8</td>
<td>21.1</td>
<td>-2.3</td>
</tr>
<tr>
<td>Greece</td>
<td>11.9</td>
<td>20.7</td>
<td>18.9</td>
<td>9.2</td>
<td>18.9</td>
<td>15.6</td>
<td>-7.0</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>37.9</td>
<td>25.0</td>
<td>54.5</td>
<td>43.0</td>
<td>54.5</td>
<td>34.6</td>
<td>-16.7</td>
</tr>
<tr>
<td>Denmark</td>
<td>8.7</td>
<td>11.8</td>
<td>15.8</td>
<td>7.6</td>
<td>15.8</td>
<td>11.4</td>
<td>-7.1</td>
</tr>
<tr>
<td>Hungary</td>
<td>18.6</td>
<td>25.4</td>
<td>22.8</td>
<td>12.5</td>
<td>22.8</td>
<td>21.0</td>
<td>-4.2</td>
</tr>
<tr>
<td>Austria</td>
<td>13.7</td>
<td>15.3</td>
<td>21.7</td>
<td>6.0</td>
<td>21.7</td>
<td>12.1</td>
<td>-8.0</td>
</tr>
<tr>
<td>Ukraine</td>
<td>6.2</td>
<td>11.0</td>
<td>2.4</td>
<td>-6.0</td>
<td>2.4</td>
<td>4.9</td>
<td>3.8</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>3.3</td>
<td>2.7</td>
<td>2.9</td>
<td>-6.1</td>
<td>2.9</td>
<td>2.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>15.6</td>
<td>13.7</td>
<td>21.6</td>
<td>9.9</td>
<td>21.6</td>
<td>16.6</td>
<td>-6.1</td>
</tr>
<tr>
<td>Germany</td>
<td>9.6</td>
<td>14.5</td>
<td>15.1</td>
<td>1.4</td>
<td>15.1</td>
<td>8.7</td>
<td>-5.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>12.3</td>
<td>19.7</td>
<td>22.3</td>
<td>9.5</td>
<td>22.3</td>
<td>12.8</td>
<td>-10.0</td>
</tr>
<tr>
<td>Italy</td>
<td>8.9</td>
<td>15.4</td>
<td>15.8</td>
<td>1.5</td>
<td>15.8</td>
<td>9.3</td>
<td>-6.9</td>
</tr>
<tr>
<td>France</td>
<td>12.5</td>
<td>18.3</td>
<td>20.9</td>
<td>7.4</td>
<td>20.9</td>
<td>12.6</td>
<td>-8.4</td>
</tr>
</tbody>
</table>

| Correlation       | 1.0 | 0.9 | 1.0 | 0.5 | 1.0 | 0.3 |
| Mean              | 9.6 | 15.8| 15.0| 6.4 | 15.0| 16.6| 5.4 |
| Standard Deviation| 19.0| 19.2| 21.1| 15.3| 21.1| 11.5| 5.8 |
Some more results

Chart 2. Marginal Contribution of component level price substitutions on the 1996-2000 SNA growth rates

-60%
-40%
-20%
0%
20%
40%
60%

Belarus
Kyrgyzstan
Azerbaijan
Ukraine
Czech Republic
Netherlands
Germany
United Kingdom
Italy
France
Republic of Moldova
Ireland
Kazakhstan
Lithuania
Croatia
Slovenia
Finland
Bulgaria
Sweden
Poland
Grease
Luxembourg
Denmark
Hungary
Austria
Azerbaijan
Ukraine
Croatia
Slovenia
Bulgaria

Actual individual consumption
Collective consumption of general government
Gross Fixed Capital Formation
Changes in inventories & acq. less disp. of val.
Exports of goods & services
Imports of goods & services
Conclusions

- Formal, comprehensive, decomposition theory feasible.
- Prices play a greater role than weights in ECP 2000
- Actual Individual consumption: pervasive inconsistency
- Other expenditure components: country-localized
- Inconsistencies neutral regarding SNA or ECP as cause
  - Further cross sectional or time series follow-up required
- Tracing source of inconsistencies useful for:
  - Flagging need for corrective action
  - Country-differentiated quality assessments
Conclusions: caveats and future work

- Weighting may play a larger role below the six component level.
- The EKS adjustment may explain some price inconsistencies.
- Testing the effect of National Account revisions not yet done.
- Not all countries were analysed because of SNA data limitations.
- Inventory change PPP had to be estimated residually for ECP 2000.