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COST OF CAPITAL SERVICES AND THE NATIONAL ACCOUNTS

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EXECUTIVE SUMMARY

1. Background

1. The topic of capital services has been discussed in various Canberra II Group meetings, including those of October 2003, April 2004, September 2004 and April 2005. It has also been discussed in meetings in Eurostat, at the OECD national accounts meeting and at the AEG meeting in December 2004. At its latest meeting in April 2005, the Canberra II Group supported the recommendations in the paper by Schreyer, Diewert and Harrison (2005) with some minor modifications.

2. The main messages to emerge from all of the meetings listed above is clear; few countries at present have a sufficiently detailed capital stock database to present robust capital service figures and even of those which do, not all are yet ready to publish these data as part of the main national accounts. However, the various groups have recognized that this is an area of keen interest to analysts and several more countries are in the process of developing capital service figures, albeit on an experimental basis. Given these circumstances, the Canberra II Group has come to the view that capital services should be included in the national accounts, but not the core accounts, and that a comprehensive description of their role and relationship with other capital measures should be provided. Nonetheless, those countries which feel that their estimates of capital services are sufficiently robust should feel free to include them in the core accounts.

3. This paper sets out to explain what capital services are, how they relate to other capital measures and how they can be estimated. Very importantly, it explains how capital stocks, capital services and depreciation should be compiled in a fully integrated way to ensure full coherence in the accounts. It also addresses criticisms concerning the validity of the capital services approach. Inevitably, it is a long paper, and excluding the Annex it runs to 20 pages. It is proposed that substantial parts of the paper be used in the updated SNA and the revised OECD manual on Measures of Capital.

2. Issue and recommendations

3. In a production process, labour, capital and intermediate inputs are combined to produce output. Conceptually, there are many facets of capital input that bear a direct analogy to labour input. Capital goods are seen as carriers of capital services that constitute the actual input in the production process. For

purposes of productivity and production analysis, then, capital services constitute the appropriate measure of capital input.

4. Consumption of fixed capital is sometimes thought of as reflecting the full benefits or costs of using fixed assets. That this is a misconception can easily be shown by taking the case where fixed assets are not owned by a firm but rented from another unit who owns the capital good. The price the owner charges for the rental comprises depreciation (consumption of fixed capital), a return reflecting either financing costs or the opportunity cost of holding capital and there may be an item reflecting changes in the market price of the asset (e.g. when an asset is expected to lose value quickly, this has to be factored into the rental).

5. If all fixed assets were leased on the market, rental values would be directly observable and national accountants could turn to these data to estimate the cost of capital services. In practice, many fixed assets are owned by their users and no rental transactions can be observed. To estimate the cost of capital services to owner-users, an imputation has to be made that brings together the various elements of rentals as described above. As often, imputing unobserved values raises conceptual and empirical issues and one objective of the present document is to provide guidance on the choice of these elements.

6. At present, the national accounts provide no measure of the value, price or volume of capital services. There is no explicit link between capital stock and value added except the entry of consumption of fixed capital to explain the difference between gross value added and net value added. Yet it has always been recognised that operating surplus is income deriving from the use of capital in production just as compensation of employees is income deriving from the use of labour. There is increasing interest in exploring exactly how different levels and types of capital stock influence the level of operating surplus. This has led to increased attention being paid to the (previously) academic interest in capital services because of its application to productivity studies.

7. Whereas the introduction of costs of capital services into the accounts is of interest in itself, they should also be internally consistent with measures of the net capital stock so that the volume and price measures of capital services, depreciation and net income aggregates in the national accounts as well as balance sheets form a coherent entity. This will also allow researchers and statistical offices to produce consistent indicators of multi-factor productivity (see OECD (2001a)) which are of significant analytical interest.

8. There is thus a more general objective to develop a set of data that integrates new measures of capital services with more traditional measures of the net and gross capital stock, depreciation and net measures of income and production. These considerations have led the Canberra II Group to adopt the following recommendation

Recommendation 1: capital services should be introduced into the national accounts, but not the core accounts (unless a country feels that their estimates are good enough to do so) and the SNA should describe the concept of capital services and their role in contributing to production. This description should explain the relationship between capital services and other capital-related data and encourage statistical offices to create a consistent and transparent set of capital-related data that serves both the analysis of income and wealth and the analysis of production and productivity. There is no proposal to change the basic structure of the accounts.

9. If all fixed assets were leased on the market, rental values would be directly observable and national accountants could turn to these data to estimate the cost of capital services. In practice, many fixed assets are owned by their users and no rental transactions can be observed. To estimate the costs of capital services to owner-users, an imputation has to be made that brings together the various elements of rentals. One objective of the paper is to provide guidance on the choice of these elements. Hence, much of the main body of the paper is devoted to the computation and interpretation of user costs measures.

10. Methodological choices have to be made in computing user costs, and one of them relates to selecting the rate of return, which constitutes an important element of user costs. The basic choice lies between an exogenous rate (such as a specific interest rate) or an endogenous rate (calculated using the observed remuneration of capital). If the latter is chosen it is necessary to estimate that part of mixed income which is attributable to capital. With an endogenous rate of return, estimates of capital services are exactly equal to gross operating surplus *plus* the capital component of gross mixed income.

11. The Canberra II Group recommends the following:

Recommendation 2:

- ***The final choice between an endogenous and an exogenous rate of return is left to the implementing statistical office. However, an exogenous, ex-ante measure for the rate of return should be associated with an ex-ante measure for depreciation and price changes;***

- ***it is preferable that user costs be formulated in terms of real rates of return, i.e., to treat rates of return and price changes jointly as spelled out in expression (10) in paragraph 34 of the main body of the paper; and***

- ***as a matter of practical importance, it is recommended that mixed income be split into capital and labour components in order to allow the formation of aggregate measures of the remuneration of labour and capital.***

12. There is agreement that fixed assets are sources of capital services. Together with the fact that there is statistical coverage of investment flows into fixed assets, they will clearly enter the scope of capital services measures. There are, however, several other assets that may play a role in the provision of capital services but:

- Some entities are at present not recognised as assets, such as research and development;

- Some entities are non-produced such as land;

- Inventories are assets that are not always included in the scope of assets that deliver capital services;

- Empirically, the measurement of some assets is very difficult – a good example being historical monuments.

13. Notwithstanding further discussions in the Canberra II Group with regard to inventories, the following recommendation is made:

Recommendation 3: In principle, capital services measures should be comprehensive in the sense that they apply to all non-financial assets, except valuables. This includes identifying capital services for inventories and land. In practice, however, there are some assets for which the usefulness of calculating capital services, for example for historical monuments, is questionable or at least a less immediate priority than for most fixed assets.

14. The Canberra II Group encourages countries who so wish, who feel their estimates are robust enough and the interest in the results strong enough, to include the estimates of capital services as an "of which" entry in the standard national accounts tables. One place for this might be the production account but given the centrality of this account and the capital services link to operating surplus, a possibly better location, at least for current price estimates, would be in the generation of income account where operating surplus is shown. However, because elements of the generation of income account are not amenable to constant price estimation it is much simpler to present constant price estimates of capital services in the production account.

15. For other countries interested in developing capital service estimates, the proposal is to present the estimates in a supplementary table, but one which is consistent with measures of consumption of fixed capital and net and gross stock that appear in the core of the 1993 SNA.

16. The presentation of capital services, whether in the main accounts or a supplementary table, requires that operating surplus and mixed income are presented in gross, not in net terms. There is also the question of whether the measurement of non-market output includes a return to capital or simply an estimate of consumption of fixed capital. If the latter, a full presentation of capital services requires that supplementary estimates are also made for the assets used in non-market production. Whatever the specific presentational form, two steps are required:

- First, the total value of non-labour income needs to be computed by adding up gross operating surplus and the part of mixed income that is not considered compensation for labour input of self-employed persons.

- Second, when an exogenous rate of return is used the resulting measure for non-labour income can then be broken down into the value of capital services, differentiated by type of asset and a residual. An exogenous rate should also be used to estimate the return to assets used in non-market production. With an endogenous rate, the value of non-labour income exactly matches the value of capital services. However, if no operating surplus has been estimated for the assets used in non-market production and if the capital stock figure used includes these assets, the resulting endogenous rate will be artificially low.

17. Within the value of capital services, the value of depreciation (consumption of fixed capital) can be identified. Finally, net operating surplus is obtained by deducting depreciation from gross operating surplus.

Account II.1.1: [Supplementary] Generation of income account

Current SNA

Uses	Resources
D.1 Compensation of employees	B.1 *Value added*
D.2 Taxes on production & imports	
D.3 Subsidies	
B.2/B.3 *Operating surplus/mixed income*	

Proposal

Uses	Resources
D.1 Compensation of employees	B.1 *Value added*
D.2 Taxes on production & imports	
D.3 Subsidies	
B.2/B.3 *Gross operating surplus/gross mixed income*	
Gross operating surplus plus the capital component of gross mixed income ***Of which:*** ***Capital services from fixed assets used in market production*** ***-of which cfc for these assets*** ***Capital services from fixed assets used in non-market production*** ***of which consumption of fixed capital for these assets*** ***Capital services from subsoil and other natural assets*** ***Capital services from non-produced land*** ***Capital services from inventories***	

Note to the table: If no return to capital has been estimated for assets used in non-market production, this item will be exactly equal to consumption of fixed capital for those assets. In that case it would be useful to include the capital services for these items as a memorandum item. If necessary this could be estimated using the ratio of capital services to consumption of fixed capital for assets used in market production applied to the figure for consumption of fixed capital for assets used in non-market production. Alternatively, a real rate of return specific to non-market producers can be applied.

Issues for discussion

18. Do you agree that capital services should be introduced in the national accounts? If so, should they be in the core accounts or supplementary accounts?

19. Do you agree that capital services, depreciation and capital stock measures should be compiled in an integrated and consistent manner, and that a comprehensive description should be included the SNA?

20. Do you agree with the proposed formulae for the estimation of capital services and the options given for the rate of return?

21. Do you agree that all non-financial assets, except valuables, produce capital services and should be included in the scope of capital services where feasible?

COST OF CAPITAL SERVICES AND THE NATIONAL ACCOUNTS

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1. Introduction

22. At its meetings in October 2003 and April 2004, the 'Canberra II Group' on Non-financial Assets discussed a paper by Ahmad (2004), proposing the introduction of capital services measures into the SNA. While the Group agreed that identifying capital services was useful from a national accounts perspective, the group also requested specific advice as to the calculation of capital services. Furthermore, several other topics have been tackled by the Canberra II Group which cannot be seen in isolation from the treatment of capital services. These are the question of depreciation and obsolescence, the treatment of the valuation of assets used in non-market production and the capitalisation of R&D. At its meeting in September 2004 in London, the Group discussed a first version of the present document whose purpose was to present a method of computing capital services that is consistent with other decisions that have been/might be made by the Group. The first version of this document listed the issues that needed settling and made several methodological recommendations without, however, working out every methodological detail.

23. This second draft takes account of comments made by countries and members of the Group, and some of the recommendations have been modified. In particular, there is no explicit recommendation to give preference to a methodology based on exogenous rates of return. Furthermore, in response to discussions at the OECD National Accounts Experts meeting in October 2004 and at the AEG meeting in December 2004, the recommendation to make the production account the central place where to show measures of capital services has been replaced by a recommendation to produce measures of capital services in a supplementary account, but leaving open the option for countries to accompany this by an entry in the production account.

24. The proposals made in this paper imply no radical changes to the presentation of the accounts or to the general meaning given to any of its aggregates, such as net operating surplus. This prudence reflects the fact that the development and understanding of statistics in this area is still relatively new, and the fact that the valuation of concepts, such as capital services, are, to some extent, dependent on assumptions about the way the economy works. Generally, the introduction of capital services into the national accounts does not change the value of the aggregates as capital services are shown in satellite accounts or, if countries so wish, as 'of which' items in the production account. Only to the extent that a consistent framework that links capital services, capital stocks, depreciation and balance sheets leads to modifications in the existing practice of countries' calculation procedures, may there be an effect on major aggregates such as GDP or NDP. One specific item, however, may change GDP and NDP estimates: the introduction

of cost of capital services for assets used in non-market production. For further reference, see Harrison (2004).

2. Capital stock and the national accounts

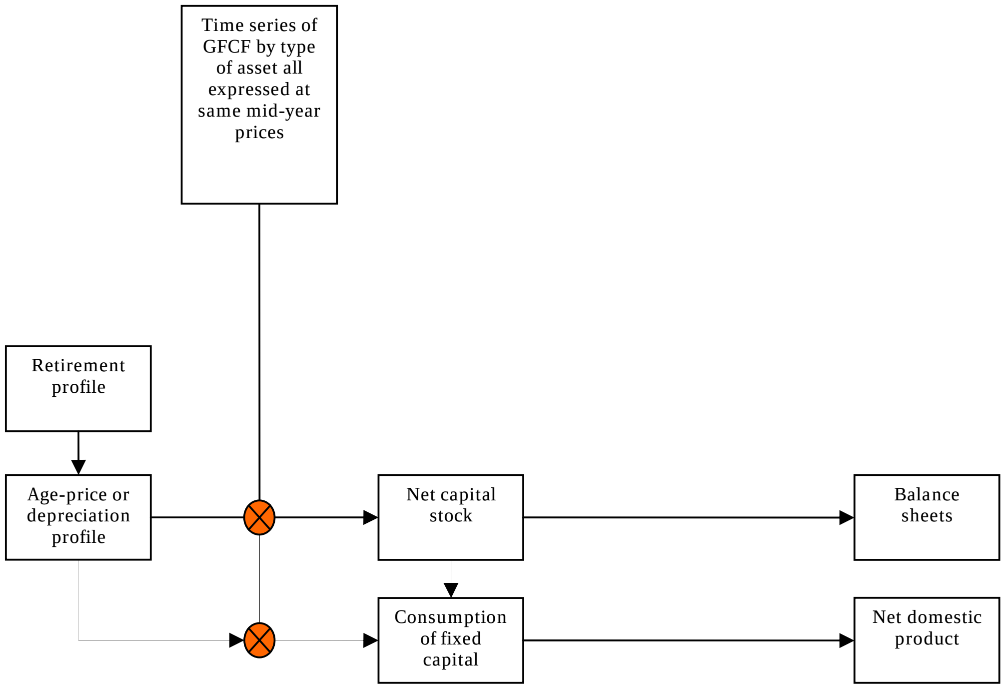
25. Capital stock features in two places in the SNA. It is needed to compile the balance sheets and it is needed to derive an estimate of consumption of fixed capital. Not all countries do compile estimates of capital stock and some use very crude methods to approximate a value of consumption of fixed capital using only flow data. However, there is no question that the SNA suggests that this is not what is recommended and there is increasing interest in having better estimates of both capital stock and consumption of fixed capital. Many of the arguments about whether an economy is sustainable or not depend on measures of income which must exclude consumption of fixed capital.

26. How is capital stock estimated? Basically by cumulating GFCF year by year and deducting retirements. Because it makes no sense to aggregate expenditures undertaken in different years without adjusting for the difference in prices between those years, all capital stock figures are in “constant prices”. These prices may be the prices of the current year, in which case past expenditures are adjusted to the current price level or may be expressed at the prices of a given year, usually the one which is the base year for constant price national accounts.

27. Retirements are calculated by postulating a life length and depreciation is calculated by superimposing a pattern of decline in value over this time. This is called an age-price profile (see Figure 1). The relevant factor for each cohort of assets is applied so that the aggregate stock figure reflects both the chosen price level and also the fact that similar assets of different ages have different values. A further complication is that apparently similar assets of different ages often incorporate improvements as compared with earlier models. Thus adjusting for prices has to incorporate adjusting for quality change also.

28. Once net capital stock figures on a consistent basis exist for two successive years, it is possible to calculate the difference between them and after deducting new investment and allowing for disposals, this is what appears as the estimate of consumption of fixed capital as currently recommended in the SNA.

Figure 1: Capital stocks and consumption of fixed capital in the SNA



3. Capital as an input into production

29. At present, there is no explicit link between capital stock and value added except the entry of consumption of fixed capital to explain the difference between gross value added and net value added. Yet it has always been recognised that operating surplus is income deriving from the use of capital in production just as compensation of employees is income deriving from the use of labour. There is increasing interest in exploring exactly how different levels and types of capital stock influence the level of operating surplus. This has led to increased attention being paid to the (previously) academic interest in capital services because of its application to productivity studies. The aim of this paper is to show that the capital service approach can be integrated with the current SNA practice of determining consumption of fixed capital in a way which does not disrupt the present system but which allows for deeper analysis and possible improvements in the underlying data on capital stock.

30. Whereas the introduction of costs of capital services into the accounts is of interest in itself, they should also be internally consistent with measures of the net capital stock so that the volume and price measures of capital services, depreciation and net income aggregates in the national accounts as well as balance sheets are fully integrated. This will also allow researchers and statistical offices to produce

consistent indicators of multi-factor productivity (see OECD (2001A)) which are of significant analytical interest.

31. An important statement of this interest in setting up integrated system of accounts, capital measures and productivity has recently been for formulated for the United States. Jorgenson and Landefeld (2004) outlined a "Blueprint for Expanded and Integrated U.S. Accounts" where they state as their '*first and foremost objective to make the NIPAs consistent with the accounts for productivity compiled by the Bureau of Labor Statistics and the flow of funds accounts constructed by the Federal Reserve Board. The boundaries of production, income and expenditures, accumulation and wealth accounts must be identical throughout the system in order to achieve consistency*'. Similar statements may well be true for other countries and have been made in the literature for many years[1].

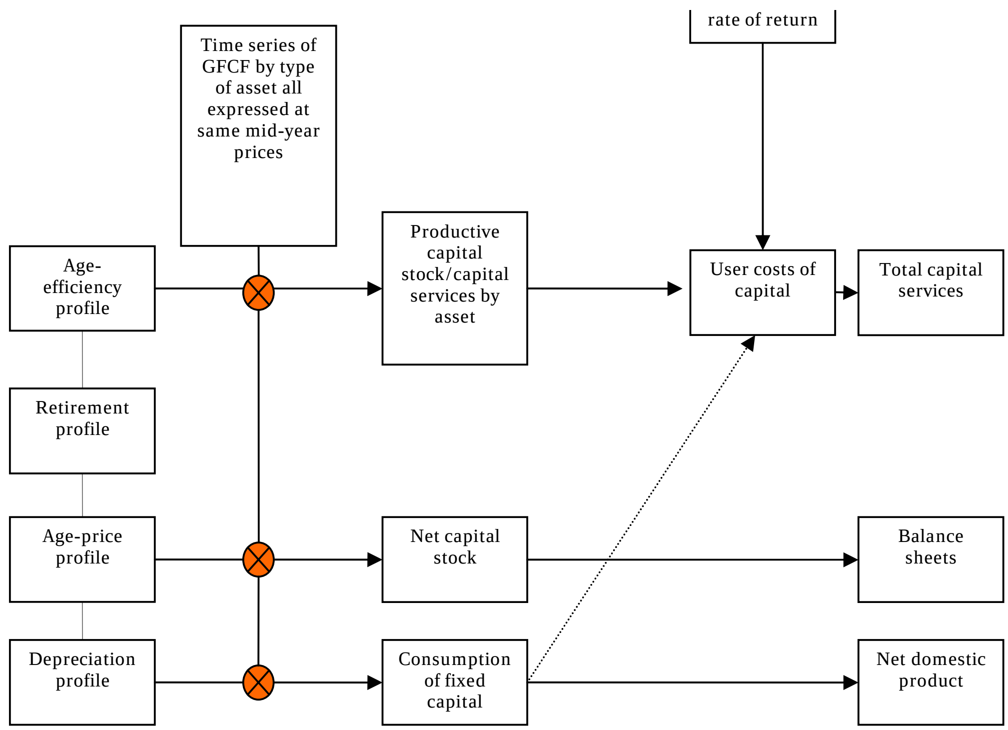
32. The value of capital stock recorded in the SNA balance sheets reflects two factors which cause the value of the asset to decline over time. One of these is that the efficiency of most assets declines over time. The second is that quite apart from a fall in price because of an efficiency decline, there is a fall in price because the useful life of the asset becomes shorter as time passes. For example a light bulb works at the same efficiency until it ceases to work at all but one would not pay the same price for an old, but still functioning light bulb as for a new one because it would not be expected to go on working for as long as the new one. This value of the capital stock is usually referred to as "wealth capital stock", sometimes "net capital stock". The value is built up by aggregating the value of all assets still in use valued at a common point in time. The sequence of parameters which is applied to each asset to derive its value at any point in time is called an "age-price profile".

33. If we wish to examine the contribution of an asset to production, we are only concerned with the efficiency decline and not with the effect of aging per se. In order to do this we build a figure for what is called "productive capital stock" by applying a parameter to each asset which reflects only the decline in efficiency. These parameters are described as age-efficiency profiles.

[1] For example, the fact that assumptions about depreciation rates, the pattern of user costs by age of asset or the pattern of asset prices by age of assets cannot be made independently of each other was first realised by Jorgenson and Griliches (1967; 257) (1972; 81-87). The algebra for switching from one method of representing capital inputs by age to another was first developed by Christensen and Jorgenson (1969; 302-305) (1973) for the geometrically declining depreciation model. The general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989) and Hulten (1990; 127-129) (1996; 152-160). Diewert and Lawrence (2000) further generalized the work of these authors by relaxing the assumption that assets of different ages were perfectly substitutable; i.e., Diewert and Lawrence worked out user costs for each asset by age and then applied normal superlative index number theory to aggregate assets of the same type over ages. The need for consistency of computation methods for capital stocks, capital services and depreciation has also been clearly stated in OECD (2001a,b).

34. It is possible to relate age-price profiles and age-efficiency profiles to each other (see annex). Thus the two measures of capital stock we have discussed are different but entirely consistent, one reflecting both the decline in efficiency and the effect of ageing (the wealth capital stock) and the other reflecting the decline in value due only to a decline in efficiency (the productive capital stock).

Figure 2: An integrated system of capital services, capital stocks and consumption of fixed capital



35. Figure 2 provides an overview of such an integrated framework. Starting from a set of investment data and constant-quality price indices, to which various parameters are applied, the integrated framework provides the following measures:

- ✓ A set of depreciation parameters, applied to a time series of past investments (consistently valued) yields a measure for the consumption of fixed capital;
- ✓ Consumption of fixed capital deducted from gross measures yields measures of net domestic product, and net value-added;
- ✓ A set of age-price parameters, applied to a time series of past investments (consistently valued) yields a measure for the net capital stock;

- ✓ A set of age-efficiency parameters applied to a time series of past investments (consistently valued) yields a measure for the productive stock for each type of asset;

- ✓ User cost weights applied to rate of change of productive stocks yields a measure for the rate of change of capital services.

36. A step-by-step discussion of the mechanics of implementing the integrated system is provided in the Annex to this document.

37. It is worth noting that all but the last two measures figure in the present 1993 SNA and are routinely computed by many countries. The shaded area shows those computations that are presently included in the national accounts.

38. Nonetheless, a true integration of capital services, stocks and depreciation measures also provides the opportunity to revisit or examine several additional issues. They include:

- ✓ the separation of mixed income of unincorporated enterprises into the contributions from labour and capital (see above)

- ✓ the measurement of the contribution of land to production (see below)

- ✓ the treatment of owner-occupied housing – if an agency decides to implement user cost and capital services measures, the treatment of owner-occupied housing in the national accounts should be consistent with this user cost methodology and this may entail a change in the measurement of the imputed rent of owner-occupied housing

- ✓ the treatment of assets used in non-market production

- ✓ the contribution of natural resources to production

- ✓ the treatment of costs of ownership transfer and costs of disposal of assets.

Recommendation 1: capital services should be introduced into the national accounts, but not the core accounts (unless a country feels that their estimates are good enough to do so) and the SNA should describe the concept of capital services and their role in contributing to production. This description should explain the relationship between capital services and other capital-related data and encourage statistical offices to create a consistent and transparent set of capital-related data that serves both the

analysis of income and wealth and the analysis of production and productivity. There is no proposal to change the basic structure of the accounts.

4. Volumes and prices of capital services

39. Capital services cannot be observed directly, so they must be estimated in much the same way as consumption of fixed capital is estimated rather than being observed. The basic assumption is that capital services are proportional to the level of productive capital stock and so factors are needed to apply to the stock levels in order to derive estimates of capital services.

40. The rental price or user cost (u^t_0)is the price per unit of capital service. It represents the cost to the owner of of tying up one unit of capital stock for use in production rather than seeking another form of return from it. Because it is common to assume that the flow of capital services is a fixed proportion of the stock of assets, the total value of capital services for a particular type of asset is obtained by multiplying the user cost term for a new asset by the productive stock of an asset of particular type. The productive stock (K^t) is built up from past investment flows by applying the age-efficiency profiles which show for each period of time what proportion of the original efficiency of the asset when new remains available. (see also annex). Thus, the value (cost) of capital services of a particular type is:

$$\text{Cost of capital services} = u_0^t K^t$$

41. Only brief mention is made here of aggregation across capital goods of different age. For a discussion of aggregation issues in this context, see Diewert (2001), Diewert and Lawrence (2000), Hulten (1990).

5. How to estimate capital services

42. In a production process, labour, capital and intermediate inputs are combined to produce output. Conceptually, there are many facets of capital input that bear a direct analogy to labour input. Capital goods are seen as carriers of capital services that constitute the actual input in the production process. For purposes of productivity and production analysis, then, capital services constitute the appropriate measure of capital input. At present, however, the national accounts provide no measure of the value, price or volume of capital services.

43. Consumption of fixed capital is sometimes assumed to reflect the full benefits or costs of using fixed assets. That this is a misconception[2] can easily be shown by considering the case where fixed assets

[2] For a fuller discussion see Triplett (1996).

are not owned by a firm but rented from another unit who owns the capital good. The price the owner charges for the rental comprises depreciation (consumption of fixed capital), a return reflecting either financing costs or the opportunity cost of holding capital and there may be an item reflecting changes in the market price of the asset (e.g., when an asset is expected to loose value quickly, this has to be factored into the rental).

44. If all fixed assets were leased on the market, rental values would be directly observable and national accountants could turn to this data to estimate the cost of capital services. In practice, many fixed assets are owned by their users and no rental transactions can be observed. To estimate the costs of capital services to owner-users, an imputation has to be made that brings together the various elements of rentals as described above. As often, imputing unobserved values raises conceptual and empirical issues and one objective of the present document is to provide guidance on the choice of these elements.

45. The idea that the production account does not explicitly identify the total values of capital services from fixed assets but instead records them within value-added or operating surplus is not, of course, new. The impetus to separately identify these capital services now however, largely reflects the increased interest in growth accounting and productivity analysis (OECD (2001), Fraumeni et al (2003), Jorgenson and Landefeld (2004)).

5.1. Interpreting and measuring rental prices or user costs

46. When rentals and the cost of capital services cannot be observed directly, the various components have to be added up to approximate the cost of capital services. A simple method for deriving a formula for the cost of using an asset during period t is the following argument. Suppose a producer purchases a new asset at the beginning of period t at a cost of P_0^t, where the subscript 0 means that the asset is 0 periods old at the time of purchase (the arguments can readily be extended to the actual or implicit purchase of used assets). At the end of period t, with "normal" usage of the asset, the producer anticipates that the asset will be worth P_1^{t+1}. The subscript 1 indicates that the asset will be 1 period old and the superscript indicates that the asset valuation is made at the end of period t or equivalently, at the beginning of period t+1. We have used italics to denote this used asset price because it is an *anticipated price* that may or may not turn out to be correct. At first glance, it would appear that the cost of buying the asset at the beginning of period t, using it for period t in a "normal" way, and then selling it at the end of period t for its anticipated value is simply $P_0^t - \mathit{P}_1^{t+1}$. However, this formulation of the user cost of capital neglects the fact that funds received at an earlier time are more valuable than funds received at a later date. In order to make the currency unit received at the end of the period, P_1^{t+1}, equivalent to the currency unit paid out at the beginning of the period, P_0^t, , it is necessary to either multiply P_0^t by $1 + r^t$ or divide P_1^{t+1} by $1 + r^t$

where r^t is the nominal cost of financial capital that the producer faces at the beginning of period t. We will divide P_1^{t+1} by $1 + r^t$ in order to obtain the following formula for the *beginning of period t user cost of capital*:

$$(1)\quad u_0^t \equiv P_0^t - P_1^{t+1}/(1 + r^t).$$

47. We have formed the user cost from the perspective of prices that prevail at the beginning of period t because we believe that this is how rental and leasing markets work: owners of assets who lease them to other users must set their rental prices for the accounting period based on information and expectations that prevail at the beginning of the leasing period. However, the reader should be able to rework our arguments using *end of period user costs*[3] of the form $P_0^t(1+r^t) - P_1^{t+1}$.

48. Although formula (1) is very simple, it is not very instructive. In particular, how exactly does the producer form expectations about what the asset will be worth at the end of the period? We will now bring depreciation and anticipated asset price inflation into the model. The *anticipated price of a used asset at the end of period t*, P_1^{t+1}, will be related to the *anticipated price of a brand new asset at the end of period t* of the same quality as the initially purchased asset, P_0^{t+1}, by the following equation:

$$(2)\quad P_1^{t+1} = (1 - \delta_0)\, P_0^{t+1}$$

where δ_0 is the *anticipated one period depreciation rate for a new asset at the start of period t+1*.[4] The anticipated price of a new asset at the start of period t+1, P_0^{t+1}, is also used in the next definition, which defines the *expected nominal period t asset price inflation rate*, i^t:

$$(3)\quad 1 + i^t \equiv P_0^{t+1}/ P_0^t.$$

49. Now substitute (2) and (3) into (1) and we obtain the following expression for the user cost of capital:[5]

$$(4)\quad u_0^t \equiv P_0^t - (1 - \delta_0)(1 + i^t)\, P_0^t /(1 + r^t)$$

$$= (1+r^t)^{-1}[(1+r^t) - (1 - \delta_0)(1 + i^t)]\, P_0^t$$

$$(5)\quad = (1+r^t)^{-1}[r^t - i^t + \delta_0(1 + i^t)]\, P_0^t.$$

[3] This is done explicitly in Diewert (2001) (2004) and in the annex to the present paper.

[4] To use Hill's (2000) terminology, this is a cross sectional depreciation rate.

[5] This method for deriving the user cost of capital (5) is essentially due to Diewert (1974; 504), except that our present formulation makes the role of expectations clearer.

50. Thus the period t user cost of capital is equal to $(1+r^t)^{-1}$ (which will usually be close to one) times the beginning of period t asset price P_0^t times a number of element comprised in the term in brackets:

- the nominal beginning of period t rate of return or opportunity cost of capital r^t less the anticipated (nominal) asset inflation rate i^t (so that $r^t - i^t$ can be loosely interpreted as an asset specific anticipated real rate of interest);

- plus the asset inflation rate adjusted anticipated depreciation rate[6] $\delta_0(1 + i^t)$.

51. Assuming that a constant quality asset price index is available so that P_0^t does not present problems in terms of implementation, it can be seen that in order to form the user cost defined by (5), it is necessary to obtain information on the relevant nominal rate of return r^t, on the anticipated nominal asset inflation rate i^t and on the anticipated depreciation rate δ_0.

52. However, formula (5) is not our "final" recommended user cost formula. Instead of working with nominal interest rates and inflation rates, it is more practical to work with real rates. After making some preliminary definitions, we show how simplified user cost formulae can be obtained.

53. Let the consumer price index for the economy at the beginning of period t be c^t and let the anticipated end of period t consumer price index be c^{t+1}. Then the anticipated general consumer inflation rate for period t is ρ^t defined by the following equation:

$$(6)\ 1 + \rho^t \equiv c^{t+1}/c^t .$$

54. The anticipated general inflation rate for period t (ρ^t) along with the beginning of period t nominal interest rate (r^t) can be used to define the period t (anticipated) *real interest rate* r^{*t} and the *period t anticipated real asset inflation rate* i^{*t} as follows:[7]

$$(7)\ 1 + r^{*t} \equiv (1 + r^t)/(1 + \rho^t);$$

$$(8)\ 1 + i^{*t} \equiv (1 + i^t)/(1 + \rho^t).$$

55. Now substitute (7) and (8) into the user cost formula (4). We find that the resulting formula simplifies to the following one:

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[6] Christensen and Jorgenson (1969; 302) derive $[r^t - i^t + \delta_0(1 + i^t)]\ P_0^t$ as the user cost of capital in a continuous time optimization model with geometric depreciation. Alternative user cost formulae were derived by Christensen and Jorgenson (1973), Jorgenson (1989; 10), Hulten (1990; 128) and Diewert and Lawrence (2000; 276).

[7] These definitions date back to Fisher (1896).

$$(9)\ u_0^t \equiv P_0^t - (1-\delta_0)(1+i^{*t})\, P_0^t/(1+r^{*t})$$

$$(10)\quad = (1+r^{*t})^{-1}[r^{*t} - i^{*t} + \delta_0(1+i^{*t})]\, P_0^t.$$

56. The formula (10) has the same general form as our earlier formula (5) but *expected real rates* have replaced expected nominal rates[8].

57. The user cost formula (10) is one of our two preferred formulae. We prefer it over the equivalent formula (5) because real rates are much more stable than nominal rates (particularly in high inflation countries). Thus, it will be easier to approximate the anticipated real rates in (10) than the nominal rates in (5). Also, real anticipated holding gains (or losses) in an asset are likely to be much smaller in magnitude than nominal expected holding gains.

58. A reasonable approximation to (10) in many circumstances can be obtained by setting the anticipated real holding gains term i^{*t} in (10) equal to zero. That is, the anticipated rise in the price of the asset is assumed to be exactly the same as the anticipated rise in the general price level. If this is done, the resulting user cost formula simplifies to:

$$(11)\ u_0^t = (1+r^{*t})^{-1}[r^{*t} + \delta_0]\, P_0^t.$$

59. Thus this *simplified no real holding gains user cost* depends only on the period t anticipated real interest rate r^{*t}, the anticipated cross sectional depreciation rate δ_0, and the beginning of period t asset purchase price. The user cost formula (11) is our second preferred user cost formula. Its main advantage over our other preferred formula (10) is that it is not necessary to estimate anticipated real holding gains and thus formula (11) is more *reproducible* than formula (10), since different investigators will have different techniques for forming expected or anticipated holding gains. At the same time, if relative asset prices show marked trends, the use of (11) may introduce a bias into the weighting structure of different assets' capital services flows (see annex).

60. Since the real interest rate r^{*t} will usually be small in magnitude, a reasonable approximation to (11) is:

$$(12)\ u_0^t = [r^{*t} + \delta_0]\, P_0^t.$$

61. This user cost formula, due essentially to Walras (1954; pp268-269), says that the user cost of capital is equal to the anticipated real interest rate plus the anticipated depreciation rate times the beginning of the period stock price of the asset.

[8] This user cost formula is broadly consistent with the approach advocated in Hill and Hill (2003).

62. An important fact to notice is that the user cost formulae (11) and (12) involve expected *real* interest rates and not *nominal* interest rates. The use of nominal rates in (11) and (12) in place of real rates could lead to tremendously inaccurate user costs in high inflation countries or in periods of high inflation.

5.2. More on the rate of return

63. Does the estimate of capital services explain gross operating surplus and the capital part of gross mixed income exactly? Or is the estimate of capital service independent so that there is another element of value-added not explained by remuneration of labour and capital? Answers to these questions determine the choice of the rate of return. The issue was first raised by Diewert[9] (1980) and then more extensively examined by Harper, Berndt and Wood (1989). There are two broad options:

- Use of an endogenous (internal) rate of return (estimated capital services exactly corresponds to gross operating surplus and the capital element of gross mixed income)
- Use of an exogenous (external) rate of return (estimated capital services is unlikely to be exactly equal to gross operating surplus and the capital element of gross mixed income)

64. The endogenous option is frequently used in empirical research. It assumes that gross operating surplus plus the capital component of mixed income exactly exhausts the costs of capital services. Given the value for costs of capital services, for the capital stock and depreciation, there is only one unknown variable, the rate of return and the equation can be solved to yield an endogenous measure of the rate of return.

65. This procedure brings with it several advantages: from a theoretical perspective, it is consistent with a fully competitive economy and production processes under constant returns to scale. From a practical viewpoint, computation is straightforward, and results can be of analytical interest in themselves. For example, it would be interesting to compare internal rates of return between industries or between countries. Finally, the fact that the costs of capital services exactly exhaust gross operating surplus plus the capital component of mixed income avoids interpreting any difference term between the value of capital services and gross operating surplus that may show up otherwise. At the same time, the choice of an endogenous rate raises at least two other questions.

9 . "Which r should be used? If the firm is a net borrower, then r should be the marginal cost of borrowing an additional dollar for one period, while if the firm is a net lender, then r should be the one period interest rate it receives on its last loan. In practice, r is taken to be either (a) an exogenous bond rate that may or may not apply to the firm under consideration, or (b) an internal rate of return. I tend to use the first alternative, while Woodland and Jorgenson and his co-workers use the second. As usual, neither alternative appears to be correct from a theoretical a priori point of view, so, again, reasonable analysts could differ on which r to use in order to construct a capital aggregate." Diewert (1980; 476-477).

66. First, the economic assumptions that are needed to justify the use of an internal rate are stringent[10] and it is not obvious that they hold empirically.

67. Second, the endogenous method cannot be applied for those institutional units for which the national accounts do not generate an independent measure of gross operating surplus, notably non-market producers.

68. We now turn to the option of selecting an exogenous rate of return. Its key advantages are (i) that it does not rely on as restrictive a set of assumption as the endogenous method. Schreyer (2004) has shown that exogenous rates are compatible with occurrences of non-observed assets, imperfect competition and non-constant returns to scale; (ii) that it can deal with government units for which there is no estimation of gross operating surplus; (iii) that it avoids importing errors from output data. But there are some additional advantages.

69. The first additional advantage is that the exogenous method permits modelling the rate as an expected or required rate[11]. If the taxes on profits are altered, for example, this has implications for what rate of return would be required after the change as compared with the rate required before the change. The second additional advantage of an exogenous rate is that it may provide a means of splitting mixed income between income to labour and income to capital. In principle, if there are independent estimates for the cost of capital services of those institutional units whose income is mixed, it is possible to sort out the share of labour and capital remuneration. Such information could be compared against plausible estimates of the labour income of self-employed. Obtaining the empirical information on capital stocks and capital services by institutional unit may be difficult but at least there is a possibility of advancing on the analysis of mixed income.

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[10] The set of assets has to be complete in the sense that all assets are observed by the statistician who compiles the national accounts. This is far from obvious. The national accounts provide no indication as to exactly which factor of production is remunerated through gross operating surplus. Fixed assets are certainly among them but they are not necessarily the only ones. The business literature offers a wealth of discussions about the importance of intangible assets, and there are good reasons to argue that such assets account at least for part of gross operating surplus. If an endogenous rate is computed on the basis of those fixed assets that are measured in the accounts, but if there are other, unmeasured assets that provide capital services, the resulting rate is liable to bias. Perfect foresight has to prevail so that the ex-post rate of return on each asset (implicitly observed by the national accountant as part of GOS) equals its ex-ante rate return, the economically relevant part in the user costs of capital services. There has to be absence of residual profits (or losses) that may arise in the presence of market power, under non-constant returns to scale or with publicly available capital assets.

[11] There is thus no assumption of perfect foresight and this helps to deal with the question of expectations: the level of capital services is what the entrepreneur expects when making decisions about the use of assets in production. If the costs of capital services turn out to be less than gross operating surplus, the entrepreneur has made some pure profit or some of the gross operating surplus pertains to non-measured assets. Further, when the exogenous rate is an expected rate, it reflects the conditions (in particular the implicit rental prices) that producers are facing when deciding about production and investment. Also, from a purely practical perspective, if there are implausibly large differences between the estimated cost of capital services and gross operating surplus or if the latter is persistently lower than the former, this may be an indication of data problems in the accounts and provide useful insights to statisticians. For example, Diewert and Lawrence, in a recent paper, used industry-level data for Australia and found a number of implausible results for industry-level endogenous rates of return. This may reflect data issues rather than economic reality.

70. However, there are also several disadvantages to the exogenous model. First, and foremost, a choice has to be made as to exactly which rate should be chosen – options are manifold with potentially important impacts on results. There is also a question whether the rate should be allowed to vary between industries or sectors, and if so, to which statistical source the national accountant should turn for this purpose.

71. Second, there may be occurrences of economically meaningless negative user cost. Equation 10 shows that a negative user cost will result if the expected nominal cost of financial capital is lower than the expected nominal inflation rate. But if these expectations materialised, there would be a question of why the asset owner would continue to hold onto it since there would be no economical rationale for doing so. , As explained by Harper, Berndt and Wood (1989) negative rental prices tend to occur when ex-ante exogenous rates of return are combined with ex-post rates of asset price change. It is thus important that the different components of the user cost term be treated consistently either as ex-ante or as ex-post variables.

72. Overall, thus, it would appear that although in the authors' view there are many advantages of using an exogenous, expected rate of return, a good case can also be made for using an endogenous, ex-post rate of return and this leads to

Recommendation 2:

- ***The final choice between an endogenous and an exogenous rate of return is left to the implementing statistical office. However, an exogenous, ex-ante measure for the rate of return should be associated with an ex-ante measure for depreciation and price changes;***

- ***it is preferable that user costs be formulated in terms of real rates of return, i.e., to treat rates of return and price changes jointly as spelled out in expression (10) in paragraph 34 of the main body of the paper; and***

- ***as a matter of practical importance, it is recommended that mixed income be split into capital and labour components in order to allow the formation of aggregate measures of the remuneration of labour and capital.***

6. What about constant prices?

73. One advantage of deriving an explicit estimate for part or all of gross operating surplus is the opportunity to derive a matching constant price estimate. Even though some more detailed questions about

aggregation across quantities of past investments may arise in the process of computation, the split of the value of capital services into a price and a volume component does not pose specific difficulties. By its very nature, user costs per unit of capital are the price measure of capital services.

74. To illustrate with a simple example, take the case where the stock of a particular type of asset is computed with the perpetual inventory method. As already describes, the relevant capital stock is the 'productive stock' (see OECD (2001A), Hulten (1990)) made up of past investments that are weighted with an 'age-efficiency profile' of assets of different age[12]: If we denote the new investment in any year as I^t then, as long as each year's investment is expressed at the same constant prices, then the value of the productive capital stock, K^t can be written as:

$$(14)\ K^t = I^t + h_1 I^{t-1} + h_2 I^{t-2} + ... + h_T I^{t-T}$$

75. Implicit in the above linear formulation is that investments of different age are perfectly substitutable[13] once their relative efficiency has been scaled by the factor h_s. K is then expressed in units of the most recently acquired investment good and the value of capital services at current prices of period t is given by multiplying K by the user cost of a new asset:

$$(15)\ \text{Cost of capital services in period t at period t prices} = u_0^t K^t$$

76. It is now straight forward to express the cost of capital services at constant prices of a base year, if this is the index number procedure applied in the national accounts. For example, the value of capital services in year t at prices of the base year t_0 can be computed as:

$$(16)\ \text{Cost of capital services in period t at period } t_0 \text{ prices} = u_0^{t_0} K^t .$$

77. Volume indices of capital services – the relevant measure for capital input in productivity calculations are easily established by aggregating across different types of assets. Again, the specific index number formula applicable in this case depends on the index number formula used elsewhere in the accounts and on the analytical purpose[14].

78. For implementation by national statistical offices, several additional issues have to be considered, in particular valuation of flows at average prices of a period – this concerns the value of capital services –

[12] To keep things simple, we ignore a retirement distribution.

[13] For a more general aggregation method across investment goods of different age see Diewert and Lawrence (2000).

[14] For example, a chained Laspeyres index of capital services, obtained by aggregation across different asset types i, would read as $L^{t/t-1} = \sum_i u_{0,i}^{t-1} K_i^t \Big/ \sum_i u_{0,i}^{t-1} K_i^{t-1}$

and valuation of stocks in the balance sheets at prices at the beginning and at the end of the period. How these valuation methods hang together in applied work, is spelled out in greater detail in the Annex to this document.

7. Scope of capital services

79. There is no disagreement that fixed assets are sources of capital services. Together with the fact that there is statistical coverage of investment flows into fixed assets, they will clearly enter the scope of capital services measures. There are, however, several other assets that may play a role in the provision of capital services but:

- Some entities are at present not recognised as assets, such as research and development. Although the Canberra Group is in favour of the possibility to consider R&D expenditure as investment, a number of issues need resolving before the stock of R&D can be fully integrated into the accounts (see discussion papers on R&D in the Canberra Group, such as Pitzer (2004)).

- Some entities are non-produced such as land. These give rise to income in the form of operating surplus but in the account of the user of the asset, not of the owner (unless the owner is also the user). This is different from the treatment of produced assets which always provide income to the owner regardless of which unit is the user of the asset. The SNA does not regard placing of non-produced assets at the disposal of a producer as production in itself but an action giving rise to property income. The Canberra II Group recommended and the AEG agreed to split land into two categories, "produced" land (i.e. land improvement) and non-produced land. This has implications for the measurement of capital services because, for land improvement, capital services and consumption of fixed capital will be shown in the accounts. However, when land is rented from another unit, or when other types of natural assets are used in production by units other than their owners, gross operating surplus or gross mixed income of the user has to be sufficient to allow the user to pay a return to this asset to the owner in the form of rent

- Inventories are assets that are not always included in the scope of assets that deliver capital services although statistical agencies such as the Australian Bureau of Statistics and Statistics Canada have incorporated inventories in their set of productive stocks. One issue that has been raised is whether un-wanted inventories should be considered as delivering capital services and if not, how they can be separated from other inventories. This separation would appear very difficult empirically, and an obvious pragmatic solution, already provisionally endorsed by the Canberra Group, is to simply consider all inventories as sources of capital services.

- Empirically, the measurement of some assets is very difficult – a good example being historical monuments.

Recommendation 3: In principle, capital services measures should be comprehensive in the sense that they apply to all non-financial assets, except valuables. This includes identifying capital services for inventories and land. In practice, however, there are some assets for which the usefulness of calculating capital services, for example for historical monuments, is questionable or at least a less immediate priority than for most fixed assets.

8. Presentation of capital services in the national accounts

80. The Canberra II Group encouraged countries who so wish, who feel their estimates are robust enough and the interest in the results strong enough, to could include the estimates of capital services as an "of which" entry in the standard national accounts tables. One place for this might be the production account but given the centrality of this account and that capital services link to operating surplus, a possibly better location would be in the generation of income account where operating surplus is shown.. For other countries interested in developing capital service estimates, the proposal is to present the estimates in a supplementary table, but one which is consistent with measures of consumption of fixed capital and net and gross stock that appear in the core of the 1993 SNA.

81. The presentation of capital services, whether in the main accounts or a supplementary table, requires that operating surplus and mixed income are presented in gross, not in net terms. There is also the question of whether the measurement of non-market output includes a return to capital or simply an estimate of consumption of fixed capital. If the latter, a full presentation of capital services requires that supplementary estimates are also made for the assets used in non-market production. Whatever the specific presentational form, two steps are required:

82. First, the total value of non-labour income needs to be computed by adding up gross operating surplus and the part of mixed income that is not considered compensation for labour input of self-employed persons.

83. Second, when an exogenous rate of return is used the resulting measure for non-labour income can then be broken down into the value of capital services, differentiated by type of asset and a residual,. An exogenous rate should also be used to estimate the return to assets used in non-market production. With an endogenous rate, the value of non-labour income exactly matches the value of capital services. However, if no operating surplus has been estimated for the assets used in non-market production and if the capital stock figure used includes these assets, the resulting endogenous rate will be artificially low.

84. Within the value of capital services, the value of depreciation (consumption of fixed capital) can be identified. Finally, net operating surplus is obtained by deducting depreciation from gross operating surplus.

Account II.1: [Supplementary] Generation of income account

Current SNA

Uses	Resources
D.1 Compensation of employees	B.1 *Value added*
D.2 Taxes on production & imports	
D.3 Subsidies	
B.2/B.3 *Operating surplus/mixed income*	

Proposal

Uses	Resources
D.1 Compensation of employees	B.1 *Value added*
D.2 Taxes on production & imports	
D.3 Subsidies	
B.2/B.3 *Gross operating surplus/gross mixed income*	
Gross operating surplus plus the capital component of gross mixed income ***Of which:*** ***Capital services from fixed assets used in market production*** ***-of which cfc for these assets*** ***Capital services from fixed assets used in non-market production*** ***of which consumption of fixed capital for these assets*** ***Capital services from subsoil and other natural assets*** ***Capital services from non-produced land*** ***Capital services from inventories***	

Note to the table: If no return to capital has been estimated for assets used in non-market production, this item will be exactly equal to consumption of fixed capital for those assets. In that case it would be useful to include the capital services for these items as *a memorandum item*. If necessary this could be estimated using the ratio of capital services to consumption of fixed capital for assets used in market production applied to the figure for consumption of fixed capital for assets used in non-market production. Alternatively, a real rate of return specific to non-market producers can be applied.

9. Should capital services enter the national accounts – a resumé of the discussion

85. The earlier sections in this paper discussed some of the remaining conceptual issues associated with capital services and while they may seem daunting at times, a history of empirical economic research in the area as well as a history of implementation by some statistical offices has shown that these questions can be resolved and realistic capital services measures can be implemented even if there is always room for improvement. But alongside the issues associated with the economics of capital services measures a number of other arguments have been evoked for and against the inclusion of capital services in the main

tables of the national accounts. The present section lists these discussion points and provides some comments that reflect the view of the authors of the present paper. We identified five types of objections to the introduction of capital service measures into the national accounts:

86. *First,* "*capital services are too model-based, there are too man;, too restrictive assumptions need to be adopted in the course of the calculation. National accountants should stick to the observable facts, and minimise the number of indirect estimations"*.

- It is true that capital services are model based and may be described by some as imputations but this is no more true for capital services that it is for consumption of fixed capital.. In both cases, though, it is the value of the flow which is imputed, not the flow itself. National accountants regularly use indirect estimation methods when direct observation is not available, for example the imputed rent of owner-occupied housing and the output of non-market production. The accounts should have a basis in economic theory. Whether or not an imputation is warranted, must be judged against the demand for data which supports the theory. In the case of capital services, demand abounds.

- A well-founded, well-documented and consistent estimate by official statisticians may be preferable over a myriad of lower-quality estimates by private researchers and other government departments.

87. Second, "*there are too many unsettled conceptual issues. For example, there is no agreement on whether rates of return should be chosen as exogenous, endogenous, ex-ante or ex-post variables. These things need settling before they can be taken up in the national accounts"*.

- This is true but the same holds for other important variables in the accounts, such as the use of net present values for assets when direct values are not availabel. Empirical research has shown that, although the effects of selecting an exogenous rather than an endogenous rate are not negligible, they tend to be relatively limited. As long as choices are transparent and well-documented, they should not confuse users.

88. Third, "*the Cambridge debate has never been resolved and this is the reason why capital services should not enter the accounts or: by showing the value of capital services, do statisticians pass value judgements about the distribution of factor incomes?"*

89. The Cambridge debate took place in the 1960s and 70s between the Universities of Cambridge UK and Cambridge Massachusetts (for a recent overview see Cohen and Harcourt 2003). One of the

central arguments is about the existence of capital aggregates: (i) aggregation across assets of different age or different types requires valuing them at market prices; (ii) market prices for capital goods depend on the rate of return because they are the discounted stream of future rentals; (iii) the rate of return depends itself on the wealth capital stock; (iv) this implies a circular argument and the rate of return and consequently measures of capital remuneration become arbitrary.

90. It is difficult to argue with this and how much importance one attaches to the argument is in itself a ‘political’ position. However:

- Irving Fisher, as early as 1907, pointed out that the interest rate could be viewed as the outcome of simultaneous equations

- If accepted, the argument would preclude the inclusion of any sort of capital stock in the accounts. The argument is no reason to accept the idea of capital stock but reject that of the associated capital services.

91. Fourth, "*statistical offices already find it difficult to implement traditional measures of capital. Implementing capital services measures would be a further drain on statistical offices' resources*".

92. It is correct that setting up an integrated system of capital measures is not a trivial task. It requires resources and energy to set up and the results have to be communicated to users. It may also bring out areas (such as the treatment of owner-occupied housing, see section 4) that need reviewing if they are to fit into an integrated system.

- At the same time, there are few statistical information requirements above and beyond what would be needed to put in place good-quality measures of capital stock and depreciation.

- Given the hands-on experience in some statistical offices (ABS, Statistics Canada, BLS), not every country has to start from scratch

- The annex to the present paper spells out, in considerable detail, how such measures can be implemented. While the annex may look complex at first glance, this is more a reflection of the fact that it spells out two options for developing a consistent system and is careful to take account of national accounting conventions such as average-period valuation for flows and end-of period valuation for stocks. This makes notation more complex but not the contents.

93. Fifth, "*capital services are acceptable but they should not be shown in the main accounts*"

94. The reason for this position is that it is felt that capital services would be given undue prominence by showing them there, in particular in light of their analytical nature.

- Recommendation 3 gives no specific indication as to where exactly capital services measures should be placed in the accounts. The choice is left to countries – if they so wish, capital services can be shown as an “of which” item in one of the main accounts, otherwise, capital services can be shown in a separate analytical table.

ANNEX: COMPUTING A CONSISTENT SET OF MEASURES OF CAPITAL SERVICES, NET CAPITAL STOCK AND CONSUMPTION OF FIXED CAPITAL

0.1 This annex describes how measures of capital services, depreciation, net and gross stocks can be calculated, which assumptions and which data series are needed. This annex first sets out the terminology used and then continues with a systematic description of the computation of an integrated system of capital measures.

1. Terminology

0.2 We use the following terminology:

- the *vintage* of an asset should be understood as a particular model of a class of assets or a bundle of characteristics that first appeared at one particular historical date;
- the *age* of an asset is a measure of its past usage where time is used as the metric for measuring usage. Age is not necessarily the same as vintage: there could be a car of 1999 vintage that is new (of age zero) in 2000;
- *obsolescence* is the process whereby a capital good goes out of use, out of date or experiences a decline in its capacity to generate returns for reasons other than wear and tear and catastrophes;
- *depreciation* is the loss in value of an asset as a result of physical deterioration, normal rates of obsolescence and accidental damage. Note that accidental damage is not further considered in the present document. Consumption of fixed capital is synonymous to depreciation.
- a *constant quality price index* is an index that follows the evolution of the price of a (capital) good where the price comparisons relate to capital goods with the same quality characteristics;
- the *term inflation* is used to depict the change in a general price index such as the GDP deflator or the consumer price index. We avoid the expression 'asset price inflation' which is sometimes used to depict the price change of a single type of asset;
- *the service life* is the time span during which an asset is used productively. When there is a cohort of a particular type of asset, a distinction can be made between the average service life and the maximum service life: the former shows the age of maximum retirement probability for an asset, the latter is the age at which all assets of a particular cohort will have retired.

2. Data requirements

0.3 At a minimum, the computation of capital services, net capital stocks, consumptions of fixed capital requires the following data:

- Time series of GFCF, disaggregated by type of asset and by industry at current and constant prices

- Average length of service lives of each type of asset

- Retirement distributions, i.e. information about the probability of an asset being discarded after a certain number of years of service

- Assumptions or empirical information about depreciation, i.e., about the loss in value of an asset due to wear and tear, and normal obsolescence

3. Two avenues for implementation

0.4 In what follows, two alternative ways of computing net capital stocks, depreciation and capital services are presented. Both are conceptually correct and both have been implemented in practice and there is no theoretical reason to prefer one over the other. The choice between these avenues should be governed by the availability and reliability of empirical information.

0.5 The first avenue (A) is one where the choice of parameters starts with the computation of the quantity of capital services, as expressed by an age-efficiency function. Then, the wealth aspect of capital is derived in the form of a consistent age-price function and depreciation function.

0.6 The second avenue (B) starts out with the wealth aspect of capital by way of a choice of depreciation parameters, and from there on develops consistent quantity measures of capital services by moving from age-price to age-efficiency functions.

4. Starting point: deflating investment

0.7 Whatever the specific way of implementing measures of capital services and capital stocks, the first step is the search for investment data. This is also true for the prescriptions in the SNA 1993 – and even if no attempt is made to estimate measures of capital services. Investment data should be broken down by type of asset and by economic activity. The level of dis-aggregation should be as detailed as the data allows and distinguish in particular those capital goods whose purchase prices follow different trends. Likewise, the industry break-down is important if it is believed that asset compositions vary greatly between industries and/or different industries face different purchase prices for capital goods.

0.8 The time series of current-price GFCF data are deflated by the appropriate investment price index. The investment price index should be a constant-quality price index. By applying it to investment series at historical prices, they are converted to a sequence of comparable volume estimates of investment, approximately expressed in efficiency units of the year to which the investment price index is referenced. Typically, they are the efficiency units of the latest vintage. This is important to retain because it implies that the volumes of past investment (initially expressed as physical units of the respective vintage) have now been converted into units of the latest vintage. Quality change in the class of assets is therefore reflected in the constant-price or volume measures of investment.

0.9 The investment price index between periods t and t-1 will be labelled $P_s^{i,t} / P_s^{i,t-1}$ where the superscripts denote the asset class i, and the years of purchase t and t-1. The subscript s shows the age of the investment good. The price index $P_0^{i,t} / P_0^{i,t-1}$ is a constant-quality price index[15] of new assets that

[15] Suppose that there are different models or vintages (j=1….N^i) of asset class i. A Laspeyres price index between period t and t-1 would make a comparison of a representative sample of the prices of the N models:

captures average price developments of investment goods between periods t and period t-1. Typically, this will be a chained index so that a comparison between period t and period t-s is based on the sequence $\frac{P_0^{i,t}}{P_0^{i,t-s}} = \frac{P_0^{i,t}}{P_0^{i,t-1}} \times \frac{P_0^{i,t-1}}{P_0^{i,t-2}} \times \ldots \times \frac{P_0^{i,t-s+1}}{P_0^{i,t-s}}$.

0.10 This price index is applied for deflation of investment series at historical prices to yield $I^{i,t-s} = IN^{i,t-s} / \left(P_0^{i,t-s} / P_0^{i,t}\right)$, a measure of investment expressed in (chained) prices of period t. When the resulting volume measures of investment are compared, the comparison takes place, approximately, on the basis of the most recent set of prices and in terms of efficiency units of the most recent vintage.

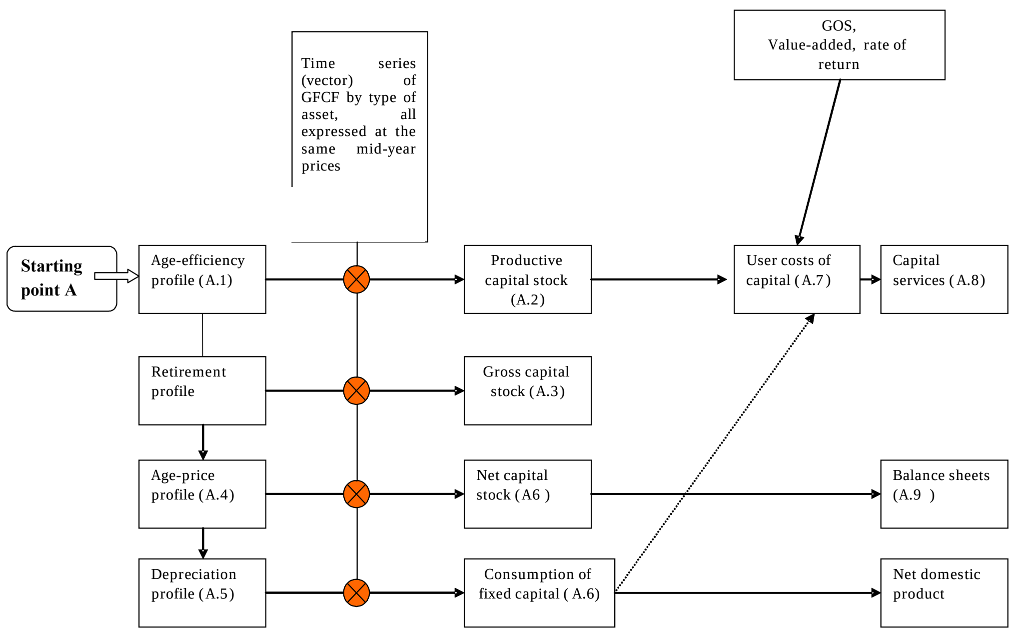
0.11 Furthermore, in line with practice in statistical offices, flows of investment are considered to be spread evenly throughout accounting periods. At the starting point of capital stock calculations one has, therefore, for every class of asset, and for every industry, a vector of present and past investment, all expressed at constant prices of a base year or expressed at chained mid-year prices of a reference year. This vector of constant price investment will be denoted as $\mathbf{I^i} = \left(I^{i,t}, I^{i,t-1}, \ldots I^{i,t-T^{i,max}}\right)$. In what follows, it will be assumed that the reference year is t, i.e., each element of the investment vector is expressed in average prices of period t.

$\frac{P_0^{i,t}}{P_0^{i,t-1}} = \frac{\sum_j {}_jP_0^{i,t} \; {}_jI^{i,t-1}}{\sum_j {}_jP_0^{i,t-1} \; {}_jI^{i,t-1}}$. This simple matched-model formulation assumes that all of the models available in base period s are also available in period t which may not be the case. Or they may be available but not representative. In such cases other methods such as hedonic techniques can help (see Triplett 2004).

A. FROM AGE-EFFICIENCY TO DEPRECIATION

0.12 This section describes avenue (A), the derivation of stock and flow measures with the quantity side of capital input (age-efficiency function) as the point of departure. Figure 3 provides an overview and reference to the sections with relevant descriptions.

Figure 3: From age-efficiency to depreciation



A.1: DETERMINING AGE-EFFICIENCY PARAMETERS

0.13 The age-efficiency profile of a single asset describes the time pattern of productive efficiency of the asset as it ages. The specific form of the age-efficiency profile is an empirical issue. The age-efficiency function should reflect losses in efficiency due to wear and tear, technical obsolescence and retirements.

0.14 The age-efficiency function for a particular asset that belongs to asset type i can be represented by g_s^i where $s = 1,...T^i$ is an index for age that runs from zero (a new asset) to T^i, the maximum service life of the asset. Because it is unlikely that all assets of the same vintage retire at the same age, T^i is a random variable, whose distribution is described by a retirement function (see below). The age-efficiency parameter is always non-negative and not larger than unity: $1 = g_0^i > g_1^i > ...g_{T^i}^i > g_{T^i+1}^i = 0$. Because the efficiency of a new asset has been set to equal one, every g_s^i represents the relative efficiency of an asset of age s compared to a new asset. In principle, the age-efficiency function can take various shapes but in practice, two functional forms have been used: a hyperbolic profile and a geometric profile.

General age-efficiency profile

0.15 Hyperbolic functions have been used by the U.S. Bureau of Labor Statistics (BLS 1983). The Australian Bureau of Statistics (ABS 2000) and OECD (Schreyer et al. 2003) adopted a similar methodology.

0.16 Hyperbolic decline takes the form:

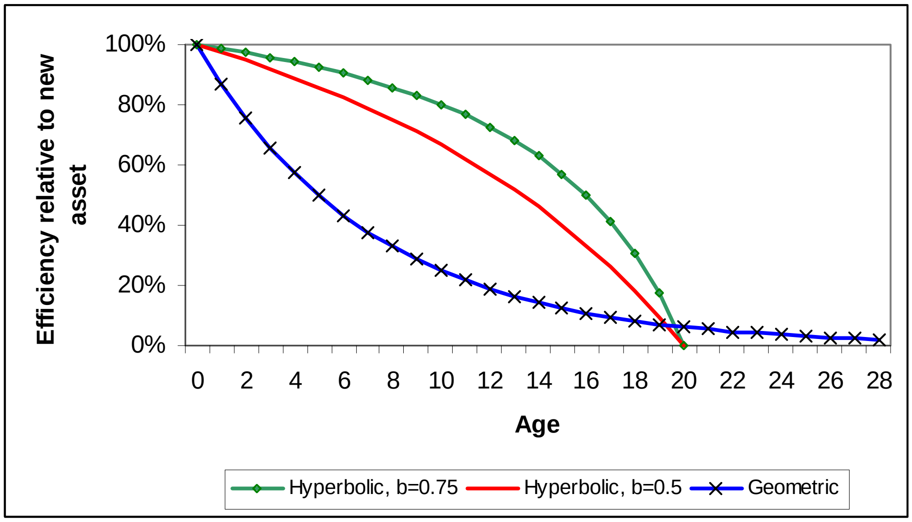
(1) $$g_s^i = \frac{T^i - s}{T^i - b^i \cdot s}$$

where g_s^i is the relative efficiency of an s-year old asset, T^i is the asset's service life, s^i is the asset's age and b is an efficiency reduction parameter. For certain parameter values, the profile can reflect an assumption that assets lose little of their productive capacity during the early stages of their service lives but experience rapid loss of productive capacity towards the final stage of their service lives.

> "The efficiency reduction parameter b is set to 0.5 for machinery and equipment and 0.75 for structures – the same parameter values as used by the BLS. The higher value for other buildings and structures redistributes efficiency decline to occur later in the asset's life, relative to machinery and equipment, the efficiency decline of which is distributed more evenly throughout the asset's life. For computer software, b is set to 0.5. For livestock, b is also set to 0.5. Clearly, a more accurate age-efficiency function and age-price function could be assumed by recognising that livestock are immature for a number of years before they begin service as mature animals. However such improvements compromise model simplicity and the improvements from doing so would be quite small. For mineral exploration, b is set to 1, implying that there is no efficiency decline in exploration knowledge. The opposite is the case for artistic originals, where be is set to 0, implying straight-line efficiency decline." (ABS 2000).

0.17 Figure 4 shows three different age-efficiency profiles. Two of them are hyperbolic functions, each with a different efficiency decline parameter. The third profile follows a geometric pattern that is further discussed below. It should be noted that the hyperbolic pattern in (1) does not necessarily yield age-efficiency profiles that are concave to the origin. Harper (1982) gives examples of hyperbolic functions that are convex to the origin.

Figure 4: Age-efficiency profiles



0.18 The presentation of the age-price profile so far has assumed that the efficiency pattern does not change over time. This may not be justified when service lives shift over time, for example as a consequence of technical or economic obsolescence. A more general approach can be adopted that introduces different age-efficiency profiles for different vintages.

0.19 The age-efficiency function above has been formulated for a single asset. When an entire cohort is concerned, account must be taken of the fact that not all assets of the same cohort will retire at the same time -there is a retirement distribution around an average service life. The retirement function describes the marginal probability of an asset of type i reaching a particular service life. Given a retirement function, an average age-efficiency function can be constructed that combines the effects of efficiency loss and retirement.

➔ See Appendix 1 for combined age-efficiency and retirement functions

0.20 When the age-efficiency and the retirement function are brought together, this gives rise to a vector of combined age efficiency-retirement parameters $\mathbf{h^i} = \left(1, h_1^i, h_2^i ...\right)$ to weight investment flows of past periods.

Geometric age-efficiency profile

0.21 The second, and empirically more frequent choice for age-efficiency profiles is a geometric pattern. It postulates that efficiency for a cohort declines at a constant rate. The concept goes at least back to Matheson (1910) although he applied it in the context of depreciation, i.e., to describe losses in value rather than efficiency (see below). Geometric efficiency profiles have been used widely by Dale Jorgenson (1995 for a collection of relevant work) and many other researchers.

0.22 Note that a geometric formulation of the age-efficiency function does not explicitly invoke a retirement function. Maximum service life is assumed to go towards infinity. The figure below shows age-efficiency functions for a cohort of assets of type i where the geometric rate has been based on a double-declining balance with an average service life of 15 years and the hyperbolic rate has been based on the same average service life, combined with a retirement distribution and a maximum service life of 20 years.

In the case of a geometric age-efficiency function, the vector of age-efficiency parameters is simply: $\boldsymbol{\delta}^{\mathbf{i}} = \left(1, (1-\delta^{i}), (1-\delta^{i})^{2}, (1-\delta^{i})^{3}, \ldots\right)$.

A.2: COMPUTING PRODUCTIVE STOCKS BY TYPE OF ASSET

0.23 The productive stock of an asset type i is derived by writing down each asset in accordance with its decline in efficiency due to age. The productive stock is computed by multiplying the vector of age-efficiency and retirement parameters by the vector of constant price investment. Because investment of past periods has been valued at average prices of period t, the productive stock is valued at average prices of period t.

Productive stock of asset type i at the end of year t (=beginning of year t+1) at average prices of year t

$$S^{i,t+1} = \begin{cases} \mathbf{h}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t'} = \sum_{\tau=0}^{T^{i}} h_{\tau}^{i} I^{i,t-\tau} \text{ for general age-efficiency profiles} \\ \boldsymbol{\delta}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t'} = \sum_{\tau=0}^{\infty} (1-\delta^{i})^{\tau} I^{i,t-\tau} \text{ for geometric age-efficiency profiles} \end{cases}$$

0.24 Productive stocks represent only an intermediate step towards the measurement of capital services. The assumption is made that the flow of capital services is proportional to the productive stock of an asset class. If the factor of proportionality is constant, the rate of change of capital services will equal the rate of change of the productive stock.

0.25 Rather than assuming that the productive stock at the beginning of a period gives rise to capital services, it would appear more realistic to assume that the average productive stock of a period gives rise to capital services. While immaterial for long-lived assets such as structures, this distinction may be of importance for short-lived assets like computers. To compute the average productive stock during period t, one starts by computing the productive stock at the beginning of period t, keeping the valuation at average prices of period t:

Productive stock of asset type i at the beginning of year t at average prices of year t

$$S^{i,t+1} = \begin{cases} \mathbf{h}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t-1'} = \sum_{\tau=0}^{T^{i}} h_{\tau}^{i} I^{i,t-1-\tau} \text{ for general age-efficiency profiles} \\ \boldsymbol{\delta}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t-1'} = \sum_{\tau=0}^{\infty} (1-\delta^{i})^{\tau} I^{i,t-1-\tau} \text{ for geometric age-efficiency profiles} \end{cases}$$

Average productive stock of asset type i during year t at average prices of year t

$$\overline{S}^{i,t} = \tfrac{1}{2}\left(S^{i,t+1} + S^{i,t}\right)$$

$$= \begin{cases} \tfrac{1}{2}\left(\mathbf{h}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t'} + \mathbf{h}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t-1'}\right) = \sum_{\tau=0}^{T^{i}} \tfrac{1}{2}\left(h_{\tau}^{i} + h_{\tau+1}^{i}\right) I^{i,t-\tau} \text{ for general age-efficiency profiles} \\ \tfrac{1}{2}\left(\boldsymbol{\delta}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t'} + \boldsymbol{\delta}^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t-1'}\right) = \sum_{\tau=0}^{\infty} (1-\delta^{i})^{\tau} (1-\delta^{i}/2) I^{i,t-\tau} \text{ for geometric age-efficiency profiles} \end{cases}$$

A.3: COMPUTING GROSS CAPITAL STOCKS BY TYPE OF ASSET

0.26 Gross capital stocks are a special case of productive stocks. Their age-efficiency profile corresponds to a 'one-hoss shay' pattern, i.e., full productive efficiency of a capital good is assumed until it retires: $g_1^i = g_2^i = \dots g_{T^i}^i = 1; g_{T^i+1}^i = 0$. As above, these age-efficiency profiles are combined with retirement profiles to yield an age-efficiency profile $\mathbf{h^i}$ that only reflects the cumulative probability of retirement for every age. Gross capital stock measures are computed by multiplying the vector of retirement parameters by the vector of investment:

Gross capital stock of asset type i at the end of year t (=beginning of year t+1) at average prices of year t

$$GKS^{i,t+1} = \mathbf{h^i} \times \mathbf{I}^{\mathbf{i},t'}$$

0.27 Because there is no explicit retirement function associated with geometric rates of age-efficiency decline, there is no gross stock defined in this case. As the gross capital stock is not a necessary ingredient for the computation of other capital measures such as capital services, this is of no importance.

0.28 Note that gross capital stocks are not, in themselves, a necessary ingredient for the computation of capital services or net capital stock measures. However, gross capital stock does figure in the 1993 SNA and is a measure that has been routinely calculated by national statistical offices, either as an intermediate step in the derivation of net capital stocks or as a variable in its own right. Similar to the general case of the productive stock, it is possible to compute a measure of the gross capital stock at the end of year t+1 as well as an average stock during the period t+1.

A.4: DERIVING AGE-PRICE PROFILES

General age-efficiency profiles

0.29 Given age-efficiency profiles, age-price profiles can be derived. The age-price profile of a class of assets shows how the value of an asset (the net capital stock) declines as it ages. In terms of a single (unique) capital good, the age-price profile is the sequence of expected market prices for this asset as it is being used in production. A fundamental relationship that determines asset prices is the condition that the price of an asset equals the discounted value of its future rentals. For a new, a one year-old and an s year-old asset, purchase prices at the beginning of period t are determined as follows:

$$\begin{aligned} &P_0^{i,t} = f_0^{i,t+1}/(1+r^t) + f_1^{i,t+2}/(1+r^t)^2 + f_2^{i,t+3}/(1+r^t)^3 + \dots \\ &P_1^{i,t+1} = f_1^{i,t+2}/(1+r^t) + f_2^{i,t+3}/(1+r^t)^2 + f_3^{i,t+4}/(1+r^t)^3 + \dots \\ &\vdots \\ &P_s^{i,t+s} = f_{s+1}^{i,t+s+1}/(1+r^t) + f_{s+2}^{i,t+s+2}/(1+r^t)^2 + f_{s+3}^{i,t+s+3}/(1+r^t)^2 + \dots \end{aligned} \tag{2}$$

0.30 In the expression above, $f_0^{i,t+1}$ is the rental price or user cost that a new asset is *expected* to fetch at the end of period t (that is, at the beginning of period t+1), $f_1^{i,t+2}$ is the rental price that the asset is expected to fetch at the end of period t+1 when it is one year old etc. In the present set-up it has been assumed that rentals are paid at the end of the period during which the asset is used. There is no compelling reason for this formulation and a rental payment at the beginning of each period is just as plausible. This issue will be taken up when user costs are discussed below. For the present discussion of age-price profiles, the issue is of no consequence. The age-price profile for asset i, is the sequence of relative prices that compares assets of different age in the same period[16] t: $1 \geq \left(\frac{P_0^{i,t}}{P_0^{i,t}}, \frac{P_1^{i,t}}{P_0^{i,t}}, \ldots \frac{P_{T^i}^{i,t}}{P_0^{i,t}} \right) \geq 0$.

0.31 With a few simplifying assumptions, age-price functions can be calculated using age-efficiency functions, a discount rate and a rate at which rental prices are expected to evolve. The latter shall be called $i^{i,t}$ and is defined as the rate of change in rental prices between the beginning and the end of a period, where the simplifying assumption has been made that the same price changes apply, independent of the age of the asset: $f_s^{i,t}(1+i^{i,t})^{\tau} = f_s^{i,t+\tau}$ for all s=0,1,2,… Another economic relationship is needed to compute the sequence of prices: producers who minimise costs of production, will use assets of different age in the same proportions as the rental prices of these assets. Put differently, the ratio of user costs for assets of different age equal the age-efficiency ratios of assets of different age: $f_s^{i,t} / f_0^{i,t} = h_s^i$. With this relationship in mind, the price of an s-year old asset relative to a new asset at the beginning of period t is:

(3) $$\frac{P_s^{i,t}}{P_0^{i,t}} = \frac{\left(h_s^i + h_{s+1}^i(1+i^{i,t})/(1+r^t) + h_{s+2}^i(1+i^{i,t})^2/(1+r^t)^2 + \ldots\right)}{\left(1 + h_1^i(1+i^{i,t})/(1+r^t) + h_2^i(1+i^{i,t})^2/(1+r^t)^2 + \ldots\right)}$$

0.32 A simple way to proceed with computations from this point onwards is to take account of the fact that $(1+r^t)/(1+i^{i,t})$ is a real interest rate (albeit one that has been deflated with an asset-specific price index) and to apply a longer-term average of real interest rates for the purpose of computing the age-price profile. For example, label the real interest rate $r^{i,t*} = (1+r^t)/(1+i^{i,t}) - 1$. This real interest rate constitutes also an element of user costs and how a real interest can be selected empirically is discussed under the section on user costs. Given a measure for the asset-specific real interest rate, the age price-profile is

(4) $$\frac{P_s^{i,t}}{P_0^{i,t}} = \frac{\left(h_s^i + h_{s+1}^i/(1+r^{*i,t}) + h_{s+2}^i/(1+r^{*i,t})^2 + \ldots\right)}{\left(1 + h_1^i/(1+r^{*i,t}) + h_2^i/(1+r^{*i,t})^2 + \ldots\right)}$$

0.33 If further simplification is sought, the real interest rate can be taken as constant, and independent of the type of asset: $r^{*i,t} = r^*$ for all i and t. Then, the age-price profile is also time-independent:

[16] The national accounts generally stipulate that flow variables such as investment or depreciation should be valued at average prices of the period to which the flow relates. Thus, if an age-price profile is used to measure flows of depreciation (see below), average prices are required. As long as the price index for new assets ($P_0^{i,t+1} / P_0^{i,t} = 1 + i^t$) equals the price index for older assets ($P_s^{i,t+1} / P_s^{i,t} = 1 + i^t$, s=1,2,..), the distinction is without importance because the price ratios of an old over a new asset at the beginning of the period will equal the same price ratio at the end of the period.

(5) $$\frac{P_s^i}{P_0^i}=\frac{\left(h_s^i+h_{s+1}^i/(1+r^*)+h_{s+2}^i/(1+r^*)^2+...\right)}{\left(1+h_1^i/(1+r^*)+h_2^i/(1+r^*)^2+...\right)}$$

Geometric age-efficiency profiles

0.34 Another important simplification arises when age-efficiency profiles are geometric. Furthermore, geometric age-efficiency profiles are defined over the entire time horizon, i.e., the maximum service life converges towards infinity. Under these circumstances, the age-price profile coincides with the age-efficiency profile[17]: $P_s^{i,t}/P_0^{i,t}=(1-\delta^i)^s$. Computationally, this is a significant simplification and many studies rely on this equality. For a description of a complete accounting system with geometric rates, see Jorgenson (1989).

A.5: DERIVING DEPRECIATION PROFILES

Depreciation profiles for general age-efficiency profiles

0.35 Depreciation is defined as the loss in value of an asset as it ages. It reflects the loss in value due to wear and tear, and foreseen obsolescence that affects the productive capacity of a capital good. Depreciation is closely associated with the notion of income, typically defined as the maximum amount that can be consumed while keeping capital intact. The measurement of depreciation depends on what exactly is meant with 'keeping capital intact'. The interpretation used here is one whereby the productive stock or the productive capacity of capital is kept intact. Then, depreciation is the outlay needed to cover the loss in value associated with wear and tear, declines in efficiency and retirement of assets. Ahmad et al. (2004) argue that this is the notion of income in the 1993 SNA. One consequence of this notion is that holding gains and losses of capital goods are not part of depreciation. A different interpretation is given by Hill and Hill (2003) who interpret 'keeping capital intact' in the sense that the ability of the capital stock to producer future income is preserved. Then, depreciation would also comprise expected stock revaluations, i.e., expected holding gains and losses.

0.36 With the age-price function for every type of asset set-up, a vector of depreciation rates per unit of past investment is readily derived. These depreciation rates are measured as $d_s^{i,t}=\left(P_s^{i,t}-P_{s+1}^{i,t}\right)/P_s^{i,t}$, i.e., as the relative loss in asset value as asset age progresses. When applied to past flows of investment, it has to be remembered that rates of depreciation apply in a cumulative way: the loss in value of a one year old investment $d_1^{i,t}I^{i,t-1}$, the loss in value of a two-year old investment is $d_2^{i,t}(1-d_1^{i,t})I^{i,t-2}$, and the depreciation of an n-year old investment is of $d_n^{i,t}(1-d_1^{i,t})(1-d_2^{i,t})\times...\times(1-d_{n-1}^{i,t})I^{i,t-n}$. Depreciation is expressed as a percentage of the value of past investments, and these rates can be brought together in a vector of depreciation parameters: $\mathbf{D}^{i,t}=\left(d_0^{i,t},d_1^{i,t}(1-d_0^{i,t}),d_2^{i,t}(1-d_1^{i,t})(1-d_0^{i,t}),...\right)$.

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[17] This can be shown as follows: $\frac{P_s^{i,t}}{P_0^{i,t}}=\frac{\sum_{\tau=0}^{T^i-s}h_{\tau+s}^i\big/(1+r^{*i,t})^\tau}{\sum_{\tau=0}^{T^i}h_\tau^i\big/(1+r^{*i,t})^\tau}=\frac{\sum_{\tau=0}^{\infty}(1-\delta^i)^{\tau+s}\big/(1+r^{*i,t})^\tau}{\sum_{\tau=0}^{\infty}(1-\delta^i)^\tau\big/(1+r^{*i,t})^\tau}=(1-\delta^i)^s=h_s^i$

Depreciation profiles for geometric age-efficiency profiles

0.37 It was shown above that when age-efficiency profiles are geometric and time and vintage invariant, age-price profiles for the same asset are also geometric. This simplifies the age-price function and the vector of depreciation coefficient that applies to past investment becomes $\boldsymbol{\Delta}^{\mathbf{i}} = \left(\delta^{i}, \delta^{i}(1-\delta^{i}), \delta^{i}(1-\delta^{i})^{2}, \delta^{i}(1-\delta^{i})^{3} \ldots\right)$

A.6: DERIVING MEASURES OF NET CAPITAL STOCKS AND CONSUMPTION OF FIXED CAPITAL

0.38 With the age-price profiles and the depreciation profiles in place, the net or wealth capital stock as well as the total value of depreciation (consumption of fixed capital) is calculated. The net or wealth stock $W^{i,t}$ represents the market value of fixed capital. It is derived by valuing past investment with the prices of a particular period t. Computationally, this is achieved by multiplying the age-price profile of a class of assets by the constant-price GFCF vector.

Net (wealth) capital stock of asset type i at the end of year t (= beginning of year t+1), at average prices of period t

$$W^{i,t+1} = p^{i,t} \times I^{i,t'} = \begin{cases} \sum_{\tau=0}^{T^{i}} \frac{P_{\tau}^{i,t}}{P_{0}^{i,t}} I^{i,t-\tau} \text{ for general age} - \text{price profile} \\ \sum_{\tau=0}^{T^{i}} (1-\delta^{i})^{\tau} I^{i,t-\tau} \text{ for geometric age} - \text{price profile} \end{cases}$$

0.39 To express the net capital stock at average prices of another year, say t*, it suffices to apply the constant-quality price index for investment goods of class i:

Net (wealth) capital stock of asset type i at the end of year t (= beginning of year t+1), at average prices of period t*=

$$W^{i,t+1} \Big/ \frac{P_{0}^{i,t}}{P_{0}^{i,t*}}$$

0.40 For certain purposes, it may be useful to measure the average net capital stock during a period. The average stock is constructed by first measuring the net capital stock at the end of period t+1, keeping the valuation at prices of period t. Then, an average is taken of the beginning and end year stock of period t+1. In the case of geometric age-price profiles, this operation turns out to be particularly simple: the end-of the year stock is 'discounted' with half the depreciation rate.

Net (wealth) capital stock of asset type i at the beginning of year t, at average prices of period t

$$W^{i,t} = p^{i,t} \times I^{i,t-1'} = \begin{cases} \sum_{\tau=0}^{T^{i}} \frac{P_{\tau}^{i,t}}{P_{0}^{i,t}} I^{i,t-1-\tau} \text{ for general age} - \text{price profile} \\ \sum_{\tau=0}^{T^{i}} (1-\delta^{i})^{i} I^{i,t-1-\tau} \text{ for geometric age} - \text{price profile} \end{cases}$$

Average net (wealth) capital stock of asset type i during period t, at average prices of period t, general age-price profile

$$\overline{W}^{i,t} = \tfrac{1}{2}\left(W^{i,t+1} + W^{i,t}\right) = \tfrac{1}{2}\left(p^{i,t} \times I^{i,t-1'} + p^{i,t} \times I^{i,t-1'}\right)$$

$$= \begin{cases} \sum_{\tau=0}^{T^i} \tfrac{1}{2}\left(\dfrac{P_{\tau+1}^{i,t}}{P_0^{i,t}} + \dfrac{P_{\tau+1}^{i,t}}{P_0^{i,t}}\right) I^{i,t-\tau} \text{ for general age} - \text{price profile} \\ (1-\delta^i/2)\sum_{\tau=0}^{T^i}(1-\delta^i)^\tau I^{i,t-\tau} = (1-\delta^i/2)W^{i,t+1} \text{ for geometric age} - \text{price profile} \end{cases}$$

0.41 The value of depreciation or consumption of fixed capital, expressed at average prices of the period t, is obtained by multiplying the vector of depreciation parameters by the vector of GFCF. Depreciation is measured at average prices of year t because the rates of depreciation are applied to past investments that are valued at average prices of year t.

Depreciation (consumption of fixed capital) of asset type i during period t, at average prices of period t, based on investment prior to period t

$$= \begin{cases} \mathbf{D^i} \times \mathbf{I}^{\mathbf{i},t-1'} \text{ for general age} - \text{price profiles} \\ \mathbf{\Delta^i} \times \mathbf{I}^{\mathbf{i},t-1'} \text{ for geometric age} - \text{price profiles} \end{cases}$$

0.42 Note that the above computation implies that depreciation during year t is calculated on the basis of investment flows prior to period t. It can be argued that depreciation should also apply to investment goods that are purchased during period t. In this case, depreciation for period t is best measured as the average of depreciation when all investment prior to t is excluded and when all investment during t is included. In the case of geometric depreciation, it is not difficult to see that average depreciation as defined above can also be expressed as a proportion of the wealth capital stock.

Depreciation (consumption of fixed capital) of asset type i during period t, at average prices of period t, based on past investment prior to period t+1

$$= \begin{cases} \mathbf{D^i} \times \mathbf{I}^{\mathbf{i},t'} \text{ for general age} - \text{price profiles} \\ \mathbf{\Delta^i} \times \mathbf{I}^{\mathbf{i},t'} \text{ for geometric age} - \text{price profiles} \end{cases}$$

Average depreciation (consumption of fixed capital) of asset type i during period t, at average prices of period t

$$= \begin{cases} \tfrac{1}{2}\left(\mathbf{D^i} \times \mathbf{I}^{\mathbf{i},t'} + \mathbf{D^i} \times \mathbf{I}^{\mathbf{i},t-1'}\right) \text{for general age} - \text{price profiles} \\ \tfrac{1}{2}\left(\mathbf{\Delta^i} \times \mathbf{I}^{\mathbf{i},t'} + \mathbf{\Delta^i} \times \mathbf{I}^{\mathbf{i},t-1'}\right) = \delta^i \overline{W}^{i,t} \text{ for geometric age} - \text{price profiles} \end{cases}$$

A.7: USER COSTS OF CAPITAL

Derivation

0.43 User costs of capital are the price that the owner-user of a capital good “pays to himself” for the service of using his own assets. Alternatively, user costs correspond to the marginal returns generated by the asset during one period of production. In a perfect market, and defining away costs of administration etc., user costs are also equal to rental prices that the owner of a capital good could achieve if he rented out the asset during one period for use in production.

0.44 User costs can be motivated in different ways. One derivation goes back to Diewert (1974, 2003) who starts from the observation that user costs are the net costs of using a capital good. To determine this cost, assume that one new unit of the capital good i is purchased at the beginning of period t at the price $P_0^{i,t}$. The ‘used’ capital good can be sold at the beginning of period t+1 at the price $P_1^{i,t+1}$. It might seem that a reasonable net cost for the use of one unit of the asset is its initial purchase price minus its end of period ‘scrap’ value. However, money received at the beginning of the period is more valuable than money received at the end of the period. Thus, in order to convert the beginning of period value into its end of period equivalent value, it is necessary to multiply $P_0^{i,t}$ by the term $(1+r^t)$, the nominal discount rate. Hence, the user cost at the end of period t for a new asset that the owner-user of the capital good faces is

(6) $$f_0^{i,t+1} = P_0^{i,t}(1+r^t) - P_1^{i,t+1}$$

0.45 A minor transformation of this expression that uses the definition of the rate of asset price change $(1+i^{i,t})P_1^{i,t} = P_1^{i,t+1}$ leads to expression (7). This same expression follows also from successive re-writing of the asset price equilibrium condition (2), showing that the intuitive derivation of the user cost formula is also compatible with the asset price equilibrium condition.

(7) $$f_0^{i,t+1} = P_0^{i,t+1}(1+r^t)/(1+i^{i,t}) - P_1^{i,t+1}$$

0.46 User costs constitute the price for the flow of capital services. In the national accounts, flow measures should be valued at average prices and an average of the user costs at the beginning and at the end of period t is formed:

(8) $$\bar{f}_0^{i,t} = \tfrac{1}{2}\left(f_0^{i,t+1} + f_0^{i,t}\right) = \bar{P}_0^{i,t}(1+r^t)/(1+i^{i,t}) - \bar{P}_1^{i,t}$$

0.47 In (8), the average user costs in period t depend on average prices in period t, defined as $\bar{P}_s^{i,t} = \tfrac{1}{2}\left(P_s^{i,t+1} + P_s^{i,t}\right)$. To this point, the assumption has been made that rentals are paid at the end of each period. This is not necessarily the case and an alternative measure of user costs could be used that assumes that rentals are paid upfront rather than after usage of the capital good. In fact, for accounting purposes, an average of the two measures would be preferable. To keep things tractable, this option has not been pursued in the body of the text. However, it is spelled out in the appendix.

➔ See Appendix 2 for user costs based on mid-period payments

0.48 A final transformation consists of using the definition of depreciation rates $d_0^{i,t} = \left(P_0^{i,t} - P_1^{i,t}\right)/P_0^{i,t}$:

(9) $$\begin{aligned}\bar{f}_0^{i,t} &= \bar{P}_0^{i,t}\left\{(1+r^t)/(1+i^{i,t})-(1-d_0^{i,t})\right\} \\ &= \bar{P}_0^{i,t}\left\{r^{*i,t}+d_0^{i,t}\right\}\end{aligned}.$$

0.49 Thus the average user cost of a new asset in period t is equal to $r^{*i,t}$, (where $(1+r^{*,i,t})=(1+r^t)/(1+i^{i,t})$ is a nominal interest rate minus an asset-specific price change which can be interpreted as an asset-specific real interest rate) times the initial costs of purchasing the asset $\bar{P}_0^{i,t-1}$ plus $d_0^{i,t}\bar{P}_0^{i,t}$, which is the value of depreciation on the asset at average prices of period t.

0.50 Above, the average productive stock of asset type i in year t, $\bar{S}^{i,t}$ has been expressed in efficiency-corrected average period t prices of a new period t capital good. The user cost price of a new period t capital good is thus the price that applies to the units that the productive stock represents. By multiplying the productive stock by the period t user costs for a new capital good, one obtains the value of capital services or the remuneration of capital services derived from asset type i:

Average value of the flow of capital services from asset type i during year t = $\bar{f}_0^{i,t}\times\bar{S}^{i,t}$

Average value of the flow of capital services from all assets during year t = $\sum_i \bar{f}_0^{i,t}\times\bar{S}^{i,t}$

Computing the rate of return

0.51 The basic considerations involved in the choice between an exogenous and an endogenous rate of return have been discussed in the main body of the text. The present discussion is thus limited to the more practical aspects of computation.

0.52 Computation is straightforward in the case of an endogenous rate of return. Non-labour income (gross operating surplus plus the capital part of mixed income) are considered compensation for capital. The endogenous rate of return is computed by solving the following equation for r:

(10) $$\text{Non-labour income} = \sum_i \bar{f}_0^{i,t}\bar{S}^{i,t} = \sum_i\left((1+r^t)/(1+i^{i,t})-(1-d_0^i)\right)\bar{P}_0^{i,t}\bar{S}^{i,t}$$

0.53 Every endogenous rate is an ex-post rate of return, based on realised variables and not on expected variables. Consequently, the remaining parts of the user cost formula should also be computed on the basis of ex-post values. In particular, the rate of asset price change and the depreciation rate should be ex-post measures.

0.54 When an exogenous rate of return is chosen, the question arises which rate(s) to select. Several considerations are necessary. First, the rate of return in the user cost expression should be consistent with the rate of return that may have been used to derive age-price profiles from age-efficiency profiles. This is irrelevant for geometric age-price and age-efficiency functions but of relevance for other profiles.

0.55 Second, the rate of return, and in fact, the entire user cost expression can be based on expected, ex-ante, or on observed, ex-post variables. There are many ways of modelling expected series and only one simple but practicable approach is presented here.

0.56 In expression (9), the first term in brackets contained two parts: a real, asset-specific rate of return $r^{*i,t}=(1+r^t)/(1+i^{i,t})-1\cong r^t-i^{i,t}$, plus the rate of depreciation $d_0^{i,t}$. Thus, if the statistician has estimates for depreciation rates, the real rate of return and the price change for the asset under consideration, then the user cost expression can be implemented.

0.57 For further simplification, it is sometimes assumed the real interest rate is asset independent ($r^{*i,t} = r^{*t}$), and one ends up with a user cost formula that is not demanding in implementation. However, when relative asset prices show markedly different trends, this simplification may entail a bias in the user cost measures and, subsequently, in the measures of capital services.

0.58 This leaves the question of the specific choice of interest rates when exogenous rates are used. For market producers, it would seem that risk-adjusted rates of return should be chosen to reflect the fact that investment in fixed capital is riskier than investment in certain financial assets. Also, in principle, both debt and equity financing should be considered. However, in practice it is difficult to obtain reliable information on the financing structure of firms. Then, an un-weighted average of relevant interest rates can provide a reasonable approximation. When user costs are computed by industry, the rates of return should also be industry-specific. An average of returns on corporate bonds, an interest rate to reflect other debt financing and possibly a measure of return on equity should be considered.

0.59 When firms invest in fixed capital they form expectations about rates of return and asset price changes and, as long as capital remains a flexible input, these expected rates of return and price change should enter the user cost evaluation. A simple method to derive such ex-ante measures is the following. On the basis of historical observations, determine a general real rate of return, i.e., a nominal rate of return deflated by some overall price index for example the GDP deflator or the CPI. Unless there is a marked trend in the historical series, it is simplest to take a long-run average across all periods and consider this long-term value constant. Alternatively, construct a smoothed series of the real interest rate. Next, again on the basis of historical price comparisons, establish a measure for the relative price change of asset i compared to the overall measure of price change (GDP deflator or CPI). For example, it may turn out that on average, computer prices have risen by 15 percentage points per year less than the prices of other goods and services. This is a relative price observation and if there are no marked trends, this is again a variable that can be set as constant. Otherwise, a time-varying but smoothed series can be constructed. When the long-run rate of relative price change is subtracted from the long-run real interest rate, one obtains a measure for a long-run (expected) asset-specific real interest rate.

Government assets

0.60 As in most countries, there is no independent estimate for gross operating surplus for non-market producers in the national accounts, there is no possibility to compute an endogenous rate of return for government assets. Thus, the statistician has to turn to the exogenous method. For government and non-market producers, the appropriate rate should be an interest rate that reflects the financing cost of government producers. An obvious choice is the rate of government bonds, based on different maturities. As before, a general price index should be selected to deflate the relevant nominal interest rates and a smoothed series of real rates should be constructed. If there are few marked trends, a simple average can serve as a reasonable approximation to expectations about real rates of return.

A.8: CAPITAL SERVICES – AGGREGATION ACROSS ASSETS

0.61 The value of capital services of a particular asset is measured as $\bar{f}_0^{i,t}\bar{S}^{i,t}$ where $\bar{f}_0^{i,t}$ - the average user cost for a new asset of type i in period t – is the price for a unit of capital services whose quantities are represented by the productive stock $\bar{S}^{i,t}$, measured at average prices of period t. An obvious way of obtaining a measure of the quantity change of all capital services is to construct a Laspeyres aggregate across different types of assets:

(11) $$L\left(\frac{S^{t+1}}{S^{t}}\right)=\frac{\sum_{i=1}^{N}\bar{f}_0^{i,t}\overline{S}^{i,t+1}}{\sum_{i=1}^{N}\bar{f}_0^{i,t}\overline{S}^{i,t}}$$

0.62 Another possibility is to construct a Paasche index:

(12) $$P\left(\frac{S^{t+1}}{S^{t}}\right)=\frac{\sum_{i=1}^{N}\bar{f}_0^{i,t+1}\overline{S}^{i,t+1}}{\sum_{i=1}^{N}\bar{f}_0^{i,t+1}\overline{S}^{i,t}}$$

0.63 A Laspeyres aggregate should be chosen if the system of national accounts uses a Laspeyres index to produce volume measures. However, it is well-known that on axiomatic and economic grounds, 'superlative indices' are preferable to the Laspeyres formula (Diewert 1976). The Fisher Ideal index belongs to the class of superlative indices and is the geometric average of the Laspeyres and Paasche index. The Fisher index number formula should certainly be chosen if the national accounts are also based on a Fisher formula for price and volume aggregates.

A.9: BALANCE SHEETS

0.64 For purposes of balance sheet entries, net stocks are required. However, balance sheets represent the market value of assets at one particular point in time – typically at the beginning and at the end of a year. This means that the relevant stocks have to be measured and valued not as average stocks at mid-year prices but as stocks at the end or at the beginning of the year and at prices at the end or at the beginning of the year.

0.65 Consider the (real) wealth stock at the beginning of the year t+1 (equivalent to the end-year stock t) $W^{i,t+1}$ that has been calculated and valued at average prices of year t. To value the same stock at prices of the end of year t, $W^{i,t+1}$ has to be multiplied through by $P_0^{i,t+1}/\overline{P}_0^{i,t}$, i.e., by the ratio between end-year prices of period t and mid-year prices of period t. This ratio can also be expressed in terms of the price index of asset i between t+1 and t, $1+i^{i,t+1}=P_0^{i,t+1}/P_0^{i,t}$:

(13) $$P_0^{i,t+1}/\overline{P}_0^{i,t}=\frac{P_0^{i,t+1}}{\frac{1}{2}(P_0^{i,t+1}+P_0^{i,t})}=\frac{1+i^{i,t+1}}{1+i^{i,t+1}/2}$$

0.66 The end-of-year t net capital stock valued at end-of –year t prices, $W_B^{i,t+1}$, is then calculated as

(14) $$W_B^{i,t+1}=\left(\frac{1+i^{i,t+1}}{1+i^{i,t+1}/2}\right)W^{i,t+1}.$$

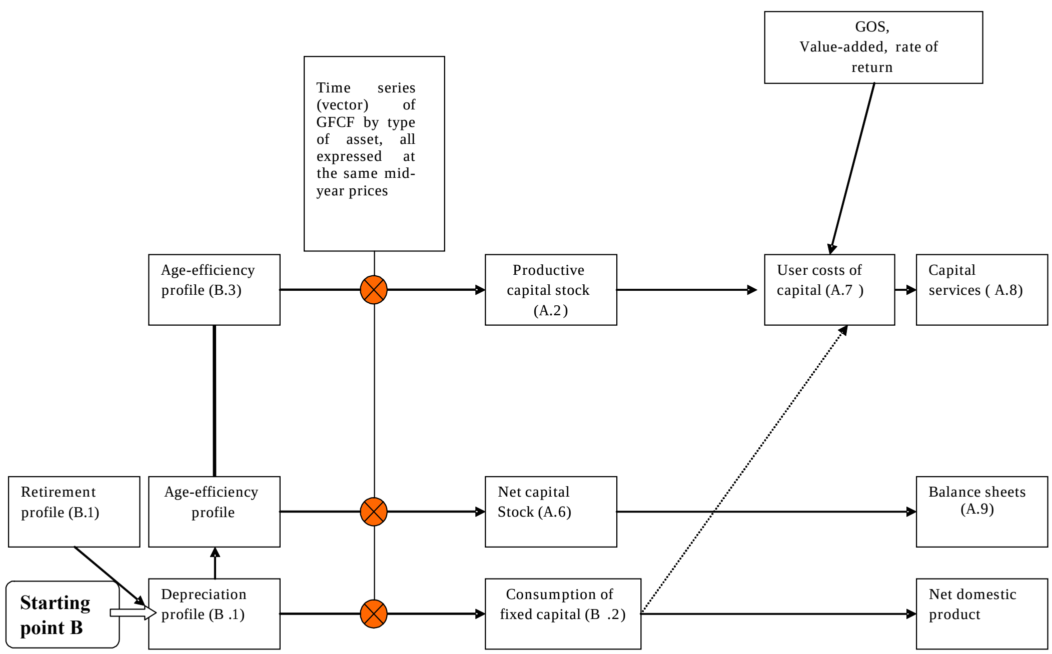
0.67 Similarly, the capital stock at the beginning of year t, valued at prices at the beginning of year t is

(15) $$W_B^{i,t}=\left(\frac{1}{1+i^{i,t+1}/2}\right)W^{i,t}.$$

OPTION B: FROM DEPRECIATION TO AGE-EFFICIENCY

0.68 This section describes the second avenue for deriving a consistent set of measures of capital services and capital stocks. It starts out with measures of depreciation and so covers ground that is familiar to statistical agencies and national income accountants. The section provides a link to well-established measures of depreciation that are based on linear age-price functions and it discusses empirical estimates of rates of depreciation. Next, it is shown how age-efficiency parameters are derived from age-price profiles – a particularly simple procedure when geometric rates of depreciation are used because for individual classes of assets, the two profiles coincide. provides an overview of this approach as well as references to the relevant sections.

Figure 5: From depreciation to age-efficiency



B.1: Determining depreciation parameters

0.69 In this document, depreciation has been defined as the loss in value of an asset due to physical deterioration (wear and tear), and due to normal obsolescence. Depreciation is a value concept, to be distinguished from quantity concepts such as the age-efficiency function that capture losses in an asset's productive efficiency. A traditional way to derive depreciation coefficients is to start from information or assumptions about assets' service lives, and make an additional assumption about the functional form of the depreciation function. In many instances, the assumption has been that depreciation follows a linear pattern. An alternative, and generally more informed way to derive depreciation parameters is by using information on used asset prices and exploit it econometrically. Consider both avenues in turn.

Straight line model of depreciation

0.70 A common model of depreciation is the straight line model. In this model, maximum service life for the durable is somehow determined, say T^i. It is then assumed that the age-price profile of the asset i follows a pattern of linear decline:

(16) $$\frac{P_s^i}{P_0^i} = \frac{T^i - s}{T^i} \text{ for } s = 0,1,..T^i$$

0.71 Successive rates of depreciation $d_s^i = \left(P_s^i - P_{s+1}^i\right)/P_s^i$ are then $d_s^i = 1/(T^i - s)$. As outlined in A.5, the total value of depreciation is obtained by multiplying a vector of past investments by a vector of depreciation parameters $\mathbf{D^i} = \left(d_0^i, d_1^i(1-d_0^i), d_2^i(1-d_1^i)(1-d_0^i),...\right)$.

0.72 When a linear pattern for the age-price or depreciation profile is assumed, no allowance is made for a retirement distribution in the computation of the profile. The retirement profile has to be built into the computation by multiplying past investment vectors through by their survival probability. This amounts to making use of the elements of the gross capital stock (see above). The total amount of depreciation for a particular period, valued at average prices of this period, is then obtained by applying the vector of depreciation parameters to the vector of past investments where each investment has been adjusted for its probability of survival. The same result is obtained by correcting the depreciation parameters by the survival probability and then multiply the resulting vector $\mathbf{DF^i} = \left(d_0^i, d_1^i(1-d_0^i)F_1^i, d_2^i(1-d_1^i)(1-d_0^i)F_2^i,...\right)$ by the unadjusted vector of past investments.

Geometric or declining balance model of depreciation

0.73 Another common model is geometric or declining balance depreciation. As mentioned above, this method is very simple computationally. The rate of geometric depreciation is given by $\delta^i = R^i / \overline{T}^i$ where $\overline{T}^i$ is the average service life and R^i is the estimated declining-balance rate. Sometimes R is chosen to equal 2 ("double declining balance") but it is preferable to turn to empirical results for the shape of the geometric depreciation pattern.

0.74 A simple method to obtain geometric efficiency coefficients is the double-declining balance method where the rate of geometric efficiency decline is given by the following expression: $\delta^i = R^i / \overline{T}^i$. $\overline{T}^i$ is the average service life and R^i is the estimated declining-balance rate. Sometimes R is chosen to equal 2 (“double declining balance”) but empirical results do not generally support that value. For example, Fraumeni (1997) reports that for the United States, R is significantly less than 2[18].

0.75 Typically, the parameters for geometric models of depreciation are derived from econometric studies of used asset prices. Although the empirical basis is not very broad, these results provide much better foundations for depreciation estimates than simple assumptions. The principles of such studies are described below.

Empirical estimates of depreciation from used asset prices

0.76 Econometric studies of depreciation use price observations on new and used assets for several periods (for a more extensive survey of depreciation studies see Jorgenson 1996). Most approaches can be traced back to the work of Hall (1971) who put forward an econometric model of vintage price functions. Major empirical work in the field was conducted by Hulten and Wykoff (1981). Examples of more recent work are Oliner (1993), Geske and Ramey (2004), and Doms et al. (2004). In simplified form, these models can be characterised as follows.

(17) $$\ln P^{s,v,t} = a + \beta D_s + \gamma D_v + \mu D_t + \varepsilon$$

0.77 Observations on prices of a particular class of assets are distinguished by the age s of the capital good, by its vintage (i.e., a particular model, described by a set of characteristics v) and by the time of purchase t. The coefficient μ in this regression will yield an estimate of the average price change of the class of assets under consideration, while controlling for the age and for the characteristics of the models in the sample. In other words, μ is an estimate for a constant-quality price index for new assets, very much the kind of price index discussed in the context of deflating investment expenditure as a first step towards constructing measures of capital stocks.

0.78 The coefficient β, attached to the age variable, represents the percentage change in prices when age moves by one unit, holding characteristics and time constant. The economic effect measured by β captures what has been labelled ‘decay’ by some authors (see Triplett 1998 for a discussion), i.e., the loss in value due to wear and tear as a capital good is used and as it ages. It is a pure age effect in the sense that it is measured while quality characteristics are held constant. β is also the parameter liable to picking up the ‘lemons’ effect, first identified by Akerlof (1970). Used assets trade at a discount when buyers cannot assess the quality of the goods offered for sale when they assume that vendors attempt to sell deficient goods. Furthermore, β reflects any age-related influence on prices that is not picked up by differences in quality characteristics and by the passage of time[19]. Finally, if old assets have different service lives from new assets, such a premium or discount will be picked up by the depreciation coefficient, even though the assumption is usually made that service lives are constant.

[18] The Bureau of Economic Analysis uses a declining balance rate of 1.65 for most machinery and equipment and a rate of 0.91 for non-residential structures, based on Hulten and Wykoff (1981).

[19] For example, there may be increasing incompatibility of older computers with the current state of IT. This is a source of obsolescence, which may be difficult to capture by the list of characteristics v and so would be reflected in the age-related depreciation coefficient. For capital goods other than computers old assets may be marked down over new ones if the old assets have been customised when new: this discount for specificity of a used asset is also expression of obsolescence.

0.79 The coefficient γ captures the effects of product characteristics, i.e., product quality on prices. Obsolescence is directly associated with product characteristics: a new model of a class of assets may have new features or more of certain characteristics than an old model and this will typically depress the price of old models even if they are physically unchanged *as such*. Because expected obsolescence is considered part of depreciation in the national accounts, the obsolescence-related effects should be reflected in measures of depreciation. However, as Oliner (1993) has shown, when investment data has been deflated with constant quality price indices – as is typically the case – only β should form the basis for empirical estimates of rates of depreciation because quality change has already been captured by the constant-quality deflator.

0.80 Retirement is the third component of depreciation that has to be considered in econometric estimates of depreciation parameters. When observations on used asset prices are used to obtain estimates of depreciation, one has to bear in mind that by concentrating on assets that were re-sold, only surviving assets enter the estimation and the effects of retirement on depreciation are insufficiently considered. Hulten and Wykoff (1981) adjust used asset observations before they apply their econometric procedure. This permits integrating the effects of survival and consequently, the resulting depreciation rates combine the effects of retirement, decay, and obsolescence.

B.2: Deriving measures of consumption of fixed capital and net capital stocks

0.81 With depreciation coefficients available – either on the basis of assumptions about the age-price profile or on the basis of econometric estimates – the computation of consumption of fixed capital and net capital stocks follows exactly the method described under A.6.

Consumption of fixed capital at average prices of period t, general age-price profile

$$= \mathbf{DF^i} \times \mathbf{I}^{\mathbf{i},t-1'} = d_0^i I^{i,t} + d_1^i(1-d_0^i)F_1^i I^{i,t-1} + d_2^i(1-d_1^i)(1-d_0^i)F_2^i I^{i,t-2} + \ldots$$

0.82 Computing the value of depreciation is particularly straight forward under geometric age-price patterns. It suffices to multiply the average wealth stock of a particular period by the depreciation rate of the asset under consideration. This operation has already been shown above.

Consumption of fixed capital at average prices of period t, geometric age-price profile

$$= \tfrac{1}{2}\left(\Delta^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t'} + \Delta^{\mathbf{i}} \times \mathbf{I}^{\mathbf{i},t-1'}\right) = \delta^i \overline{W}^{i,t}$$

B.3: Deriving age-efficiency profiles

0.83 Given depreciation rates, a discount rate and a rate of price change for each type of asset, the user asset price equilibrium conditions (2) provide again the link between age-price and age-efficiency functions. It suffices to restate the condition that the relative efficiency of an s-year and a new asset at time t (the age efficiency profile) equal the relative user costs of these assets, and to insert the formula for user costs (9):

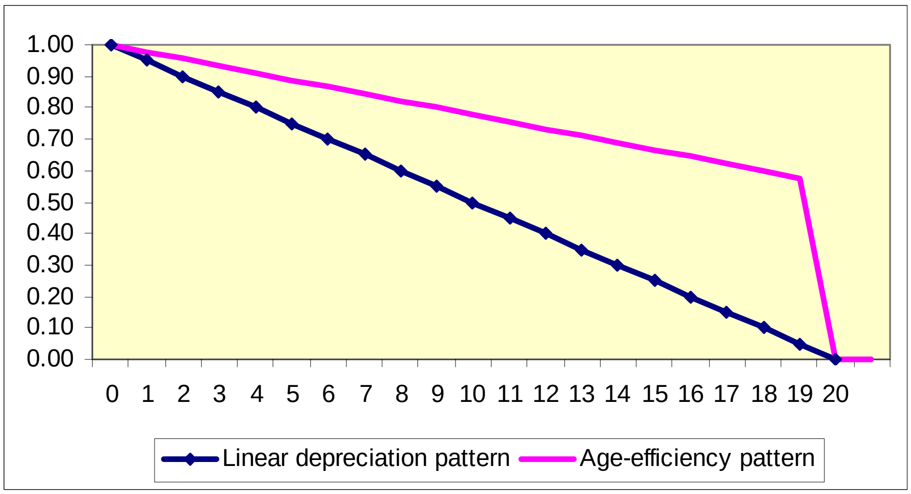
$$(18) \qquad h_s^{i,t} = \frac{\bar{f}_s^{i,t}}{\bar{f}_0^{i,t}} = \frac{\bar{P}_s^{i,t}\left\{r^{*i,t} + d_s^{i,t}\right\}}{\bar{P}_0^{i,t}\left\{r^{*i,t} + d_0^{i,t}\right\}}.$$

0.84 When depreciation is linear, the age-efficiency profile is also declining linearly with age but at a different rate than the depreciation pattern:

$$(19) \qquad h_s^{i,t} = \frac{T^i - s}{T^i} \times \frac{\left\{ r^{*i,t} + 1/(T^i - s) \right\}}{\left\{ r^{*i,t} + 1/T^i \right\}} = \frac{\left\{ (T^i - s) r^{*i,t} + 1 \right\}}{\left\{ T^i r^{*i,t} + 1 \right\}} = 1 - \frac{r^{*i,t}}{T^i r^{*i,t} + 1} \times s \,.$$

0.85 The figure below shows an example for an age-efficiency profile derived from a linear age-price profile, using the formula (19). The real interest rate for the asset was set at 4 percent and the maximum service life was taken as 20 years. It is apparent that the two profiles are different – thus, if there are good reasons to believe that depreciation follows a linear pattern, the age-efficiency profile will only be piecewise linear and in fact combine a linear and a one-hoss shay pattern.

Figure 1: An age-efficiency pattern derived from an age-price pattern



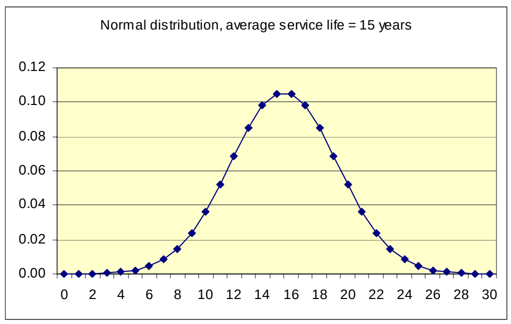
0.86 As before, computations are significantly simplified when depreciation patterns are geometric, i.e., when the value of an asset (corrected for the overall change in new asset prices) declines at a constant rate as it ages. Then, the age-efficiency profile is simply identical to the age-price or depreciation profile and no further computations are possible[20].

0.87 With age-efficiency patterns at hand, the same procedure applies for the computation of productive stocks and capital services measures as under option A (see sections A.7. and A.8.)

[20] When depreciation is geometric, the rates of depreciation are independent of the asset's age. Thus, in expression (18), the terms in brackets in the numerator and in the denominator are equal and cancel out. And the ratio of the price of an s-year old asset to a new asset is $(1-\delta^i)^s$. Consequently, the age-efficiency profile has the same form.

APPENDIX 1: COMBINED RETIREMENT AND AGE-EFFICIENCY FUNCTIONS

0.88 The retirement function is labelled $0 < F_s^i \leq 1$ where F_s^i is the marginal probability of retirement for an asset of type i with age s. Empirically, Weibull distributions, Winfrey distributions or normal distributions have been used to capture the retirement function. For a more extensive discussion of retirement functions, see OECD (2001b). To illustrate, a simple normal distribution is employed here, with an average service life of 15 years and a standard deviation of 3.75.



0.89 The age-efficiency function has been labelled $0 < g_s^i \leq 1$. It reflects the relative efficiency of an s-year old asset compared to a new asset. Hyperbolic decline takes the form $g_s^i = \frac{T^i - s}{T^i - b^i \cdot s}$. Hence, the age-efficiency parameter depends on age but also on the service life which is a random variable. The parameter b will not be further considered in this context. To generate a combined age-efficiency retirement function $0 < h_s^i \leq 1$, a value for $g_s^i(T^i)$ is computed for every s and every T. Then, every value is weighted by the marginal probability of retirement. The table below shows an extraction of the values for age 1-7 years and for T 1-15 years. Thus, each cell indicates the value of the age-efficiency function, weighted by the probability that a certain service life applies. The combined age-efficiency retirement function h_s^i is then obtained by adding the values in each column. The result is shown in the first line of the table.

Marginal probability	$\boxed{h_s \rightarrow}$	0.964	0.925	0.882	0.836	0.785	0.730	0.670
	T	1	2	3	4	5	6	7
0.0000	1	0.000						
0.0001	2	0.000	0.000					
0.0002	3	0.000	0.000	0.000				
0.0004	4	0.001	0.001	0.000	0.000			
0.0010	5	0.002	0.002	0.001	0.001	0.000		
0.0022	6	0.004	0.003	0.003	0.002	0.001	0.000	
0.0044	7	0.008	0.007	0.006	0.005	0.004	0.002	0.000
0.0083	8	0.014	0.012	0.011	0.010	0.008	0.006	0.003
0.0145	9	0.022	0.021	0.019	0.017	0.015	0.012	0.009
0.0238	10	0.034	0.032	0.030	0.027	0.024	0.021	0.017
0.0364	11	0.049	0.047	0.044	0.040	0.037	0.032	0.028
0.0518	12	0.066	0.063	0.059	0.055	0.051	0.046	0.040
0.0688	13	0.082	0.078	0.074	0.070	0.065	0.060	0.054
0.0850	14	0.094	0.090	0.086	0.082	0.077	0.071	0.065
0.0980	15	0.102	0.098	0.093	0.089	0.084	0.079	0.073
etc.					etc.			

$$\boxed{g_s(T) = \frac{T - s}{T - b \cdot s} \times F_s}$$

APPENDIX 2: USER COSTS BASED ON MID-PERIOD PAYMENTS

When rentals are paid at the beginning of each period, the asset price equilibrium condition (1) becomes

$$
(20)\qquad
\begin{aligned}
&P_0^{i,t} = u_0^{i,t} + u_1^{i,t+1}/(1+r^t) + u_2^{i,t+2}/(1+r^t)^2 + \ldots \\
&P_1^{i,t+1} = u_1^{i,t+1} + u_2^{i,t+2}/(1+r^t) + u_3^{i,t+3}/(1+r^t)^2 + \ldots \\
&\vdots \\
&P_s^{i,t+s} = u_s^{i,t+s} + u_{s+1}^{i,t+s+1}/(1+r^t) + u_{s+2}^{i,t+s+2}/(1+r^t)^2 + \ldots
\end{aligned}
$$

where the rental payable at the beginning of each period has been labelled $u_s^{i,t+\tau}$. Successive insertion in (20) produces $P_0^{i,t} = u_0^{i,t} + P_1^{i,t+1}/(1+r^t)$. Then, the user cost of capital for a new asset, based on rental payments at the beginning of each accounting period are

$$(21)\qquad u_0^{i,t} = P_0^{i,t} - P_1^{i,t+1}/(1+r^t) \quad \text{at the beginning of period t and}$$

$$(22)\qquad u_0^{i,t+1} = P_0^{i,t+1} - P_1^{i,t+2}/(1+r^t) \quad \text{at the end of period t and}$$

$$(23)\qquad \bar{u}_0^{i,t} = \tfrac{1}{2}\left(u_0^{i,t+1} + u_0^{i,t}\right) = \bar{P}_0^{i,t} - \left((1+i^{i,t})/(1+r^t)\right)\bar{P}_1^{i,t} \quad \text{at mid-year of period t.}$$

0.90 As there is no theoretical reason to prefer one formulation of the timing of rental payments over the other, it is suggested to form an arithmetic average between the user cost expression based on payments at the beginning of each accounting period and the user cost expression based on payments at the end of each accounting period which amounts to saying that rentals are paid mid-period:

$$(24)\qquad \bar{\bar{f}}_0^{i,t} = \tfrac{1}{2}\left(\bar{f}_0^{i,t} + \bar{u}_0^{i,t}\right) = \left\{\bar{P}_0^{i,t} - \left((1+i^{i,t})/(1+r^t)\right)\bar{P}_1^{i,t}\right\}\left(1+r^t/2\right) = \bar{f}_0^{i,t}\left(1+r^t/2\right)/(1+r^t).$$

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