Co-ordinate transformations

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In using a GIS, we often refer to locations using co-ordinate systems. A co-ordinate system is a set of rules that specifies how co-ordinates are assigned to points. We speak of georeferencing if we bring spatial data into a common reference system using co-ordinate transformations.

This article deals with co-ordinate transformations for spatial data. A brief overview of the theory of co-ordinate transformations is given, followed by issues concerning the application of co-ordinate transformations. The article is accompanied by a website: http://kartoweb.itc.nl/geometrics. Interested readers can find here reference notes on co-ordinate systems, reference surfaces, map projections and co-ordinate transformations. The website gives also access to a list of frequently asked questions.

Co-ordinate transformations

Co-ordinate transformations are used to bring spatial data into a common reference system. Most countries have defined their own common reference system. For example, spatial data that are related to the Universal Transverse Mercator projection system may need to be transformed to the Dutch RD system if this system is the reference system in use.

Projection change

Spatial data with co-ordinates of a known projection are normally transformed from one projection co-ordinate system to another using the forward and inverse projection equations. As illustrated in figure 1 the inverse equations of the source projection are used to transform source projection co-ordinates (system A) to geographic latitude and longitude co-ordinates. The forward equations of the target projection are used to transform the geographic co-ordinates to target projection co-ordinates (system B).

Spatial data can have co-ordinates with different underlying ellipsoids or the underlying ellipsoids have different datums. This means that, apart from different ellipsoids, the centres of the ellipsoids do not coincide (see figure 2). To relate the spatial data, one may need in such a case a so-called datum transformation. For example, spatial data that are related to the European 1950 (ED 50) datum may need to be transformed to the datum underlying the Dutch RD system (this implies the Bessel 1841 ellipsoid).

In this case the projection transformation must be combined with a datum transformation step in between as is illustrated in figure 1. The forward equations take us from some projection into geographic co-ordinates. Then we apply a datum transformation (from datum A to datum B), and finally move into another map projection.

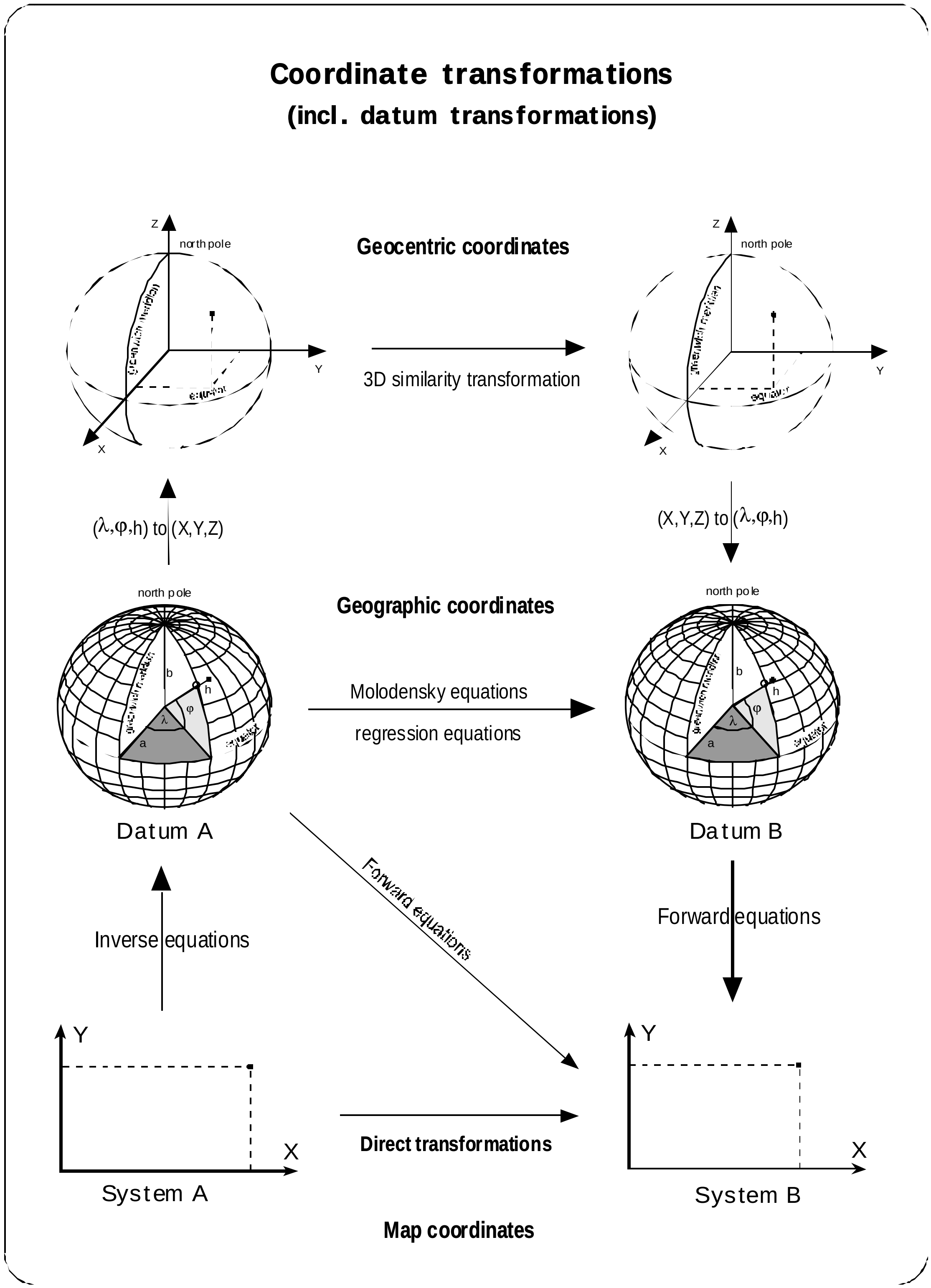

Figure 1: Overview of co-ordinate transformations

Datum transformation

Mathematically this datum transformation is feasible via 3 dimensional geocentric co-ordinates, implying a 3D similarity transformation defined by 7 parameters: 3 shifts, 3 rotations and a scale difference. These are combined with transformations between the geocentric co-ordinates and ellipsoidal latitude and longitude co-ordinates in both datum systems.

The transformation from the latitude and longitude co-ordinates into the geocentric co-ordinates is rather straightforward and turns ellipsoidal latitude (φ), longitude (λ) and height (h) into X,Y and Z, using 3 direct equations that contain the ellipsoidal parameters a and e. The inverse equations are more complicated and require either an iterative calculation of the latitude and ellipsoidal height, or it makes use of approximating equations like those of Bowring. These last have millimeter precision for ‘earth-bound’ points, i.e. points that are at most 10 km away from the ellipsoidal surface (this is the case for all topographic points).

However a good approximation of this datum transformation are the *Molodensky* and the *regression* equations, relating directly the ellipsoidal latitude and longitude, and in case of Molodensky also the height, of both datum systems (see figure 1).

The standard Molodensky equations relate ellipsoidal latitude and longitude co-ordinates and ellipsoidal height of a local geodetic datum to those of the WGS84 datum (NIMA report, 1997). The multiple regression equations relate ellipsoidal latitude and longitude co-ordinates of continental size datums to those of the WGS84 datum and involve polynomial expressions in the two ellipsoidal co-ordinates which go up to degree 9 for the time being. The main advantage of this method over Molodensky formula (implemented in most geo-software) is that better fits over continental size land areas can be achieved.

Application of projection change

Forward and inverse projection equations - as discussed earlier - are normally used to transform spatial data from one projection co-ordinate system to another. Some transformation programs, however, only include the equations that relate to a sphere as model of the earth. A spherical model can be used at small-scale but for larger scales an ellipsoid should be chosen. A spherical model assumption may result in unacceptable differences in co-ordinates.

Projection transformation programs do not always combine a projection change with a datum transformation. If one neglects the difference in datums there will be no perfect match between adjacent maps of neighbouring countries or between overlaid maps originating from different projections. It may result in differences in co-ordinates in the range of the datum shifts, which can go up to several hundred meters.

To apply the required datum transformation we need the ellipsoidal latitude, longitude and height in one datum system and the shift and rotation of the ellipsoidal axes of one datum system with respect to the other. However, datum transformation programs implemented in GIS and

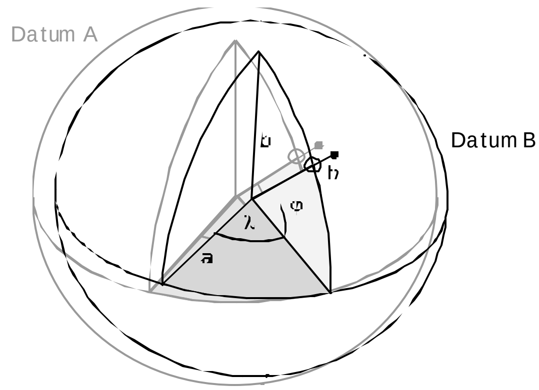

Figure 2: datum shift between two geodetic datums

Cartographic software often simplify this transformation: i.e. ellipsoidal heights (h) are taken equal to 0 or the rotation differences of the ellipsoidal axes are ignored.

Datum transformations using Molodensky equations (implemented in most geo-software) are becoming increasingly important, because of the growing use of GPS data. Very often the data is captured using the WGS84 ellipsoid and datum, and have to be transformed to a local projection with its own ellipsoid and datum. Moreover, heights measured with GPS have to be transformed to heights related to the height reference point (vertical datum) used in a particular country (this implies for the Netherlands the N.A.P.).

Direct transformations

If the underlying projection of a co-ordinate system is unknown we may relate the co-ordinate system to a known co-ordinate system on the basis of a set of selected points whose co-ordinates are known in both systems (given in figure 1 as direct transformations). These points may be ground control points or common points such as corners of houses or road intersections, as long as they have know co-ordinates in both systems.

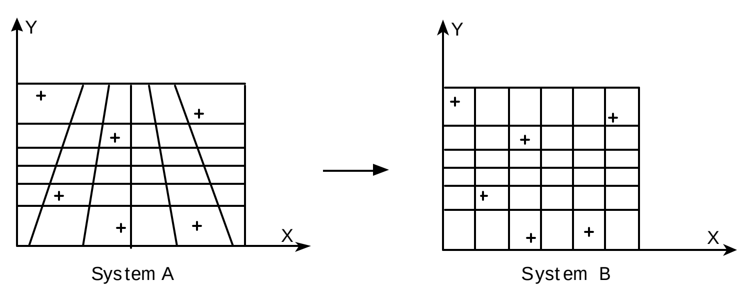

Figure 3: co-ordinate transformation performed on the basis of selected points. Here six points were chosen.

The transformation may be conformal, affine, projective, polynomial or of another type, depending on the geometric errors in the data set. These statistical two-dimensional transformations obtained by least squares adjustment have a different accuracy compared to the transformations based on projection equations. The latter take into account the earth curvature. This is especially important in the case of large areas and small scale. However, if the control points are coplanar and the extent of the area is not too large, the 2D transformation could yield a better model of co-ordinate relations than the presumed set of projection equations would do.

Image and scanned data are usually transformed by this method. Linear conformal or affine transformations can be used to rectify distortions such as a shift (or translation), a rotation or a linear scale difference. Non-linear polynomial transformations can be used to correct variable scale differences as illustrated in figure 4.

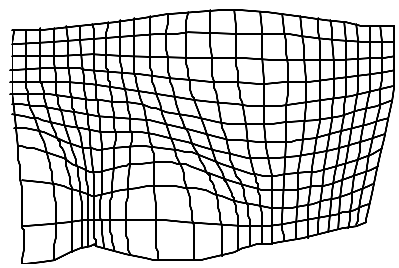

Figure 4: variable scale differences in an image

Direct transformations are also used to match vector data layers that don't fit exactly by stretching or *rubber sheeting* them over the most accurate data layer. Moreover, affine transformations are used in map digitising for the registration of a paper or scanned map.

Application of direct transformations

Different methods are in use to rectify raw images. Raw images are built up by a rectangular array of pixels with variable values, but these pixels don't have a correct geometric position yet. Co-ordinates can be assigned to the uncorrected image as is illustrated in figure 5, or the other

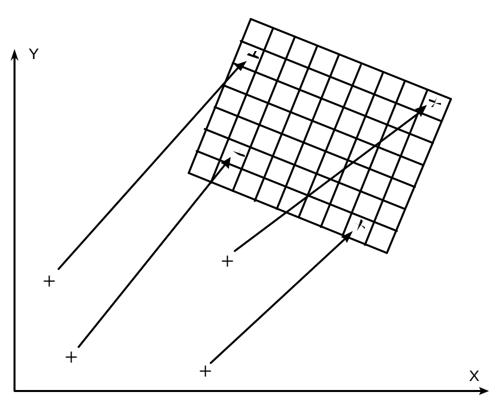

Figure 5: co-ordinate assignment: here the image is not resampled.

way round, the uncorrected image can be resampled to match it with the known co-ordinate system as is illustrated in figure 6. Resampling is a process in which for each pixel in the new co-ordinate system, a new pixel value has to be determined by means of an interpolation from surrounding pixels in the old image.

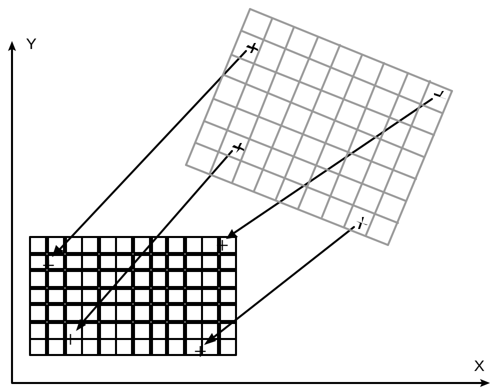

Figure 6: co-ordinate assignment: here the image is resampled

In our institute, the ITC, a frequently used GIS package is ILWIS, developed in house (website: http://itc.nl/ilwis). A typical feature with respect to co-ordinate transformations is the possibility in ILWIS to match vector and raster data by an on-the-fly transformation of the vector data. One can combine in such a way a raster image with several vector layers in one map window by dragging the vector data without the need of resampling the raster image.

The raster image can be a raw satellite image, an oblique or vertical aerial photo or a scanned topographic map. For a correct matching, one needs a set of reliable control points in the image which are linked to map co-ordinates in accordance with the method discussed earlier and illustrated in figure 5. This link is defined by a geometric correction model, for instance linear equations, projective equations, orthophoto correction equations, etc. After that, one can drag any vector map, even with another datum, over the non-corrected image.

Figure 7: Line map (cadastre en topography) as overlay on a oblique aerial photo (Enschede, North of centre, 1997)

Because there is no need to rectify (resample) the image, one can save time and disk space in case of combined analysis of raster and vector data. The degradation of the image quality due to resample errors is also avoided. Once the vector editing or analysis is completed, one can still decide to rectify the image to the well-defined co-ordinate system of the vector data.

Final comment

A common frustration for users of spatial data is the loss of co-ordinate and projection information in the process of data translation from one software program to another. To convert spatial data, a data format should be used that embeds co-ordinate and projection information. Vector data formats, however, often don't include projection information. Moreover, many raster data formats such as bitmaps (BMP) and most Tagged Image File Formats (TIFF) don't facilitate co-ordinate information.

In recent years an extension of the Tiff format, called Geo-Tiff, has been developed. Geo-Tiff files contain co-ordinates of at least two opposite corner pixels and (if applicable) also the parameters of the projection (central meridian, false origin, ellipsoid, etc.) to which the co-ordinates pertain.

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