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Towards a methodology for analyzing the coherence between PPP and SNA based estimates of GDP

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A) Introduction

An important contributor to the quality of a statistical estimate can often be the availability of another, related indicator, against which it can be checked. For example, expenditure-based GDP has income-based or industry-based GDP as points of comparison. A large difference points to a shortcoming in one or another of the estimates, and is a signal that an investigation to find the offending component(s) may be needed. If related indicators exist not only for broad aggregates, but also for detailed components, comparisons can help pinpoint the most likely problem-causing areas. In the input-output system, for example, comparing commodity supply to disposition can point to a shortcoming in one or more of the statistics measuring activity in a given commodity. Since there is one such comparison for each commodity, this opens the possibility of probing the consistency of the statistical system at several hundred points, and represents a valuable resource for quality control. This kind of consistency monitoring and analysis can be useful for assessing not only the methodology underlying a related set of statistics, but also its operational implementation during the current period. Estimates of purchasing power parity can be, and have been checked against implicit deflators and real GDP from the expenditure side of the National Accounts[2]. For example, Varjonen(2001) has made a very useful comparison of GDP per capita figures, extrapolated from the SNA, with direct estimates provided by the European Comparison Program (ECP), and traced the source of these differences at a detailed level of expenditure. This study did not break the differences down, however, into that portion due to basic data differences, and that portion due to different weighting structures, an essential distinction for analysis of consistency of the two data sets. A similar study also noted a large unexplained residual difference Varjonen(2002). These kinds of comparisons have been hampered by the lack of an exact and comprehensive model that can be used as a framework to guide them. Magnien in fact notes that a detailed investigation of the nature of the difference between real GDP growth rates, as measured by the System of National Accounts and as implied by the PPP estimates, has never been carried out in the International Comparisons Program (ICP). To fill this gap this paper develops a theoretical model of the relationship between real GDP growth rates, as measured by the System of National Accounts and as implied by the PPP estimates. Using data from the 2000 round of the European Comparison Program (ECP), it illustrates the application of this model to examine the coherence of the two sets of data.

a) The ECP 2000 round

Chart 1 illustrates the need for a formal framework for analysing the relationship between PPP-based entities and SNA-based entities. It shows real growth over the 1996-2000 period, as implied by the ECP, along-side real growth as recorded by the SNA. The ECP-implied real growth was calculated according to the following intuitive argument: if GDP in country A was 10% that in country B in 1996, and 12% in 2000, this implies that country A grew 20% more than country B over the period. Adding 20% to country B's real growth would then give a PPP-implied estimate of real growth for country A. Chart 1 is based on this calculation when Russia was taken to be the reference country B against which the others were compared.

It can rightly be objected that such a simple approach can be misleading, that it ignores the effect of changes in economic structure, relative price structures, and revisions to the National Accounts. It is important, however, to try to disentangle these various effects, since this will guide how to respond to some of the very substantial differences shown in Chart 1. If the differences are due mainly to differences in economic structures, over time and across countries, then it is a fact of life and the problem becomes one of interpretation of the numbers and their proper representation to users. If the differences, however, are mainly due to differences in the basic price data collected by the two systems, it points to the need to improve the accuracy of the collected data. We need to be able decompose the differences shown in Chart 1 into differences due to basic data differences, to revisions to accounts, to weight shifts, and to the fact that the Fisher formula in the PPP has no counterpart in the SNA. It will be especially useful if this decomposition can be made not only in total,

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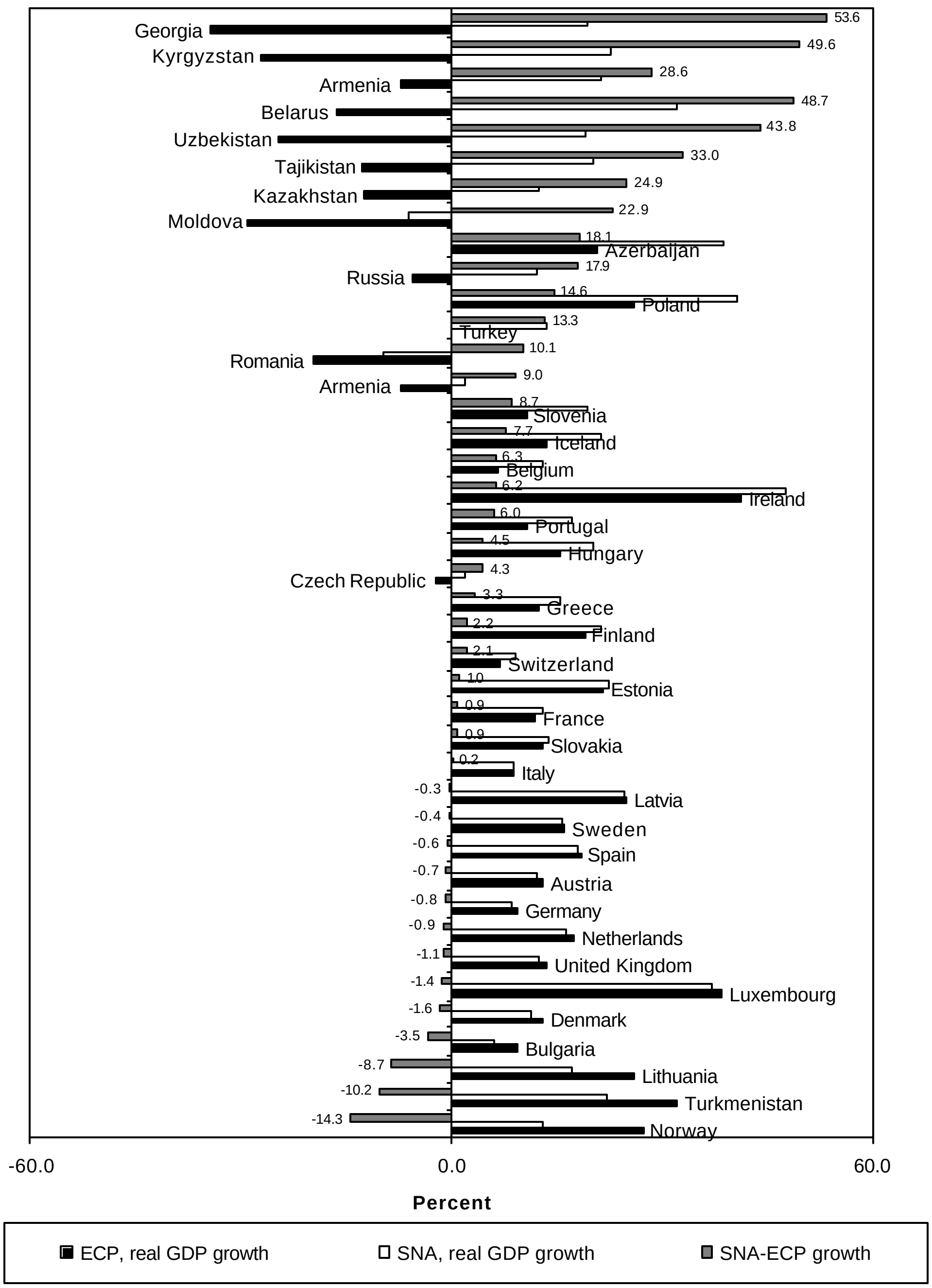
[2] Dalgaard and Sorensen(2002), Magnien(2002), Varjonen(2001, 2002)

but also for individual expenditure components. In subsequent sections we develop a framework for making such comparisons, and use it to analyse the consistency of the ECP 1996-2000 results with the SNA.

Chart 1. Comparison of ECP and SNA growth rates for real GDP (1996-2000)
(using intuitive definition of ECP implied growth)



B) Formula relating SNA and PPP-based real GDP growth

While the intuitive approach to making real growth rate comparisons illustrated in Chart 1 is too simple to be of more than limited use, it does provide a starting point for developing a more useful approach.

a) Notation

Let GDP_t^S and GDP_t^R be national accounts-based GDPs of reference and subject countries R and S respectively, valued in their own national currencies.

That is, $GDP_t^S = \sum_i p_{it}^S q_{it}^S$ and $GDP_t^R = \sum_i p_{it}^R q_{it}^R$

where i represents the i'th expenditure category.

The price ratios $\frac{p_{it}^S}{p_{it}^R}$ and $\frac{p_{it+k}^S}{p_{it+k}^R}$ will stand for PPP estimates of the i'th expenditure component. PPP estimates can be directly measured through the ECP program, or can be indirectly inferred from the Expenditure Accounts data within the System of National Accounts. To distinguish the directly measured PPPs provided by the ECP from their indirectly inferred National Accounts counterparts we introduce a tilda over the prices. Thus the price ratios $\frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R}$ and $\frac{\tilde{p}_{it+k}^S}{\tilde{p}_{it+k}^R}$ will represent component level PPPs as directly[3] measured by the ECP program, while $\frac{p_{it}^S}{p_{it}^R}$ and $\frac{p_{it+k}^S}{p_{it+k}^R}$ will represent the indirectly inferred National Accounts measures.

At the GDP level the PPP for S relative to R is given by the Fisher index plus an EKS adjustment[4] that makes the collection of GDP level indices transitive across countries.

$$PPP_t^{SR} = \left(\sum_i w_{it}^S \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} \left(\sum_i w_{it}^R \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} + e_t^{SR}$$

w_{it}^S and w_{it}^R are national currency shares in S and R respectively, for the i'th expenditure category, and e_t^{SR} represents an adjustment introduced by the combined effect of EKS and linking at the GDP level. The shares $w_{it}^S = \frac{p_{it}^S q_{it}^S}{\sum_i p_{it}^S q_{it}^S}$ and $w_{it}^R = \frac{p_{it}^R q_{it}^R}{\sum_i p_{it}^R q_{it}^R}$ may represent National Accounts estimates in current prices, either as they stood at the time the PPPs were compiled or after subsequent revisions. This distinction will be important later on.

For the moment, while developing the approach in general, we will not use the tilda notation.

GDP for S, in R's currency, is given by dividing its GDP in national currency by PPP_t^{SR}

In other words:

[3] Before EKS adjustments, and before linking.

[4] The binary Fisher indices are not transitive across countries. The EKS procedure creates an adjusted set of PPPs that are as close as possible to the original binary Fisher indices subject to the constraint that they be transitive.

$$GDP_t^{S(R)} = \Sigma_i\, p_{it}^S q_{it}^S \left[\left(\Sigma_i\, w_{it}^S \frac{p_{it}^S}{p_{it}^R} \right)^{1/2} \left(\Sigma_i\, w_{it}^R \frac{p_{it}^S}{p_{it}^R} \right)^{1/2} + e_t^{SR} \right]^{-1}$$

where of course $GDP_t^{S(R)}$ stands for S's GDP expressed in R's currency.

b) The intuitive approach in formulas

Once converted to R's currency, GDP in S can be expressed as a proportion of reference country GDP;

$$\frac{GDP_t^{S(R)}}{GDP_t^R} = \frac{\Sigma_i\, p_{it}^S q_{it}^S}{\Sigma_i\, p_{it}^R q_{it}^R} \left[\left(\Sigma_i\, w_{it}^S \frac{p_{it}^S}{p_{it}^R} \right)^{1/2} \left(\Sigma_i\, w_{it}^R \frac{p_{it}^S}{p_{it}^R} \right)^{1/2} + e_t^{SR} \right]^{-1}$$

the growth in this proportion between periods t and t+k is given by:

$$\frac{\dfrac{\Sigma_i\, p_{it+k}^S q_{it+k}^S}{\Sigma_i\, p_{it+k}^R q_{it+k}^R} \left[\left(\Sigma_i\, w_{it+k}^S \dfrac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2} \left(\Sigma_i\, w_{it+k}^R \dfrac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2} + e_{t+k}^{SR} \right]^{-1}}{\dfrac{\Sigma_i\, p_{it}^S q_{it}^S}{\Sigma_i\, p_{it}^R q_{it}^R} \left[\left(\Sigma_i\, w_{it}^S \dfrac{p_{it}^S}{p_{it}^R} \right)^{1/2} \left(\Sigma_i\, w_{it}^R \dfrac{p_{it}^S}{p_{it}^R} \right)^{1/2} + e_t^{SR} \right]^{-1}}$$

and therefore real growth of GDP in S, as implied by the PPP estimates, is given by factoring in the real [5] growth in reference county R on top of the growth in this proportion, that is by:

$$\frac{\Sigma_i\, p_{it}^R q_{it+k}^R}{\Sigma_i\, p_{it}^R q_{it}^R} \frac{\dfrac{\Sigma_i\, p_{it+k}^S q_{it+k}^S}{\Sigma_i\, p_{it+k}^R q_{it+k}^R} \left[\left(\Sigma_i\, w_{it+k}^S \dfrac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2} \left(\Sigma_i\, w_{it+k}^R \dfrac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2} + e_{t+k}^{SR} \right]^{-1}}{\dfrac{\Sigma_i\, p_{it}^S q_{it}^S}{\Sigma_i\, p_{it}^R q_{it}^R} \left[\left(\Sigma_i\, w_{it}^S \dfrac{p_{it}^S}{p_{it}^R} \right)^{1/2} \left(\Sigma_i\, w_{it}^R \dfrac{p_{it}^S}{p_{it}^R} \right)^{1/2} + e_t^{SR} \right]^{-1}} \qquad (1)$$

Multiplying and dividing by the real[5] growth in S between periods t and t+k, and rearranging terms we get:

$$INT_{t+k} = \frac{\Sigma_i\, p_{it}^S q_{it+k}^S}{\Sigma_i\, p_{it}^S q_{it}^S} \frac{\dfrac{\Sigma_i\, p_{it+k}^S q_{it+k}^S}{\Sigma_i\, p_{it}^S q_{it+k}^S}}{\dfrac{\Sigma_i\, p_{it+k}^R q_{it+k}^R}{\Sigma_i\, p_{it}^R q_{it+k}^R}} \frac{\left[\left(\Sigma_i\, w_{it+k}^S \dfrac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2} \left(\Sigma_i\, w_{it+k}^R \dfrac{p_{it+k}^S}{p_{it+k}^R} \right)^{1/2} + e_{t+k}^{SR} \right]^{-1}}{\left[\left(\Sigma_i\, w_{it}^S \dfrac{p_{it}^S}{p_{it}^R} \right)^{1/2} \left(\Sigma_i\, w_{it}^R \dfrac{p_{it}^S}{p_{it}^R} \right)^{1/2} + e_t^{SR} \right]^{-1}}$$

We have labelled this INT_{t+k} to indicate it represents the intuitive concept shown in Chart 1. The intuitive growth rate equals the real growth rate in S, times the ratio of the implicit deflator in S relative to R, divided by the ratio of PPPs at time t+k relative to time t.

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[5] For the time being we assume that constant price estimates of GDP in R and S are calculated in prices of a fixed base year, specifically of time period t. Later we will generalize the argument to other cases.

Note that if the PPP is estimated by the ratio of implicit deflators, as is often done for the purpose of projecting PPPs between benchmarks, then INT_{t+k} will exactly equal real growth in S. This gives us some confidence that INT_{t+k} is a sensible concept despite the problems associated with its interpretation when actual benchmark PPPs are used in its calculation instead of deflator-based projections.

The difference between intuitive ECP-implied growth INT_{t+k} and the SNA-based growth $\frac{\Sigma_i p_{it}^S q_{it+k}^S}{\Sigma_i p_{it}^S q_{it}^S}$ is encapsulated in the ratio of implicit deflators divided by the ratio of PPPs.

$$\frac{\dfrac{\Sigma_i p_{it+k}^S q_{it+k}^S}{\Sigma_i p_{it}^S q_{it+k}^S}}{\dfrac{\Sigma_i p_{it+k}^R q_{it+k}^R}{\Sigma_i p_{it}^R q_{it+k}^R}} \quad \frac{\left[\left(\Sigma_i \tilde{w}_{it+k}^S \dfrac{\tilde{p}_{it+k}^S}{\tilde{p}_{it+k}^R}\right)^{1/2} \left(\Sigma_i \tilde{w}_{it+k}^R \dfrac{\tilde{p}_{it+k}^S}{\tilde{p}_{it+k}^R}\right)^{1/2} + e_{t+k}^{SR}\right]^{-1}}{\left[\left(\Sigma_i \tilde{w}_{it}^S \dfrac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R}\right)^{1/2} \left(\Sigma_i \tilde{w}_{it}^R \dfrac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R}\right)^{1/2} + e_t^{SR}\right]^{-1}} \qquad (2)$$

Thus, if we can explain why (in terms of differences in weights and relative prices) the ratio of implicit deflators differs from the ratio of PPPs, we will also explain why ECP implied real growth differs from SNA-based growth.

c) *The ratio of implicit deflators as a function of SNA-based PPPs at the component level*

In order to explain why the ratio of PPPs differs from the ratio of implicit deflators, it would be useful if both were expressed as a function of the component level PPPs $\frac{p_{it}^S}{p_{it}^R}$ and $\frac{p_{it+k}^S}{p_{it+k}^R}$. The PPP is already in this form (with the exception of the EKS term), but the ratio of implicit deflators is not, so it would be useful if we can convert it to this form.

The ratio of implicit deflators can be transformed as follows:

$$\frac{\dfrac{\Sigma_i p_{it+k}^S q_{it+k}^S}{\Sigma_i p_{it}^S q_{it+k}^S}}{\dfrac{\Sigma_i p_{it+k}^R q_{it+k}^R}{\Sigma_i p_{it}^R q_{it+k}^R}} = \frac{\dfrac{\Sigma_i p_{it+k}^S q_{it+k}^S}{\Sigma_i p_{it+k}^R q_{it+k}^R}}{\dfrac{\Sigma_i p_{it}^S q_{it+k}^S}{\Sigma_i p_{it}^R q_{it+k}^R}} = \frac{\dfrac{\Sigma_i p_{it+k}^S q_{it+k}^S \dfrac{p_{it+k}^R}{p_{it+k}^R} \dfrac{q_{it+k}^R}{q_{it+k}^R}}{\Sigma_i p_{it+k}^R q_{it+k}^R}}{\dfrac{\Sigma_i p_{it}^S q_{it+k}^S \dfrac{p_{it}^R}{p_{it}^R} \dfrac{q_{it+k}^R}{q_{it+k}^R}}{\Sigma_i p_{it}^R q_{it+k}^R}} = \frac{\Sigma_i w_{it+k}^R \dfrac{q_{it+k}^S}{q_{it+k}^R}\left(\dfrac{p_{it+k}^S}{p_{it+k}^R}\right)}{\Sigma_i w_{it+k}^{R(t)} \dfrac{q_{it+k}^S}{q_{it+k}^R}\left(\dfrac{p_{it}^S}{p_{it}^R}\right)} \qquad (3)$$

where $w_{it+k}^{R(t)}$ represents constant price shares $\frac{p_{it}^R q_{it+k}^R}{\Sigma_i p_{it}^R q_{it+k}^R}$ for time t+k, valued in prices of time t.

This is a similar functional form as the PPP, in that it is a function of the price ratios $\frac{p_{it}^{S}}{p_{it}^{R}}$ and $\frac{p_{it+k}^{S}}{p_{it+k}^{R}}$, but the composite weights $w_{it+k}^{R}\frac{q_{it+k}^{S}}{q_{it+k}^{R}}$ and $w_{it+k}^{R(t)}\frac{q_{it+k}^{S}}{q_{it+k}^{R}}$ are for the moment unknown, and we need them if we are to use this approach to compare ECP and SNA-based data. Of course the shares w_{it+k}^{R} and $w_{it+k}^{R(t)}$ can be calculated from the SNA data, but we also need to be able to calculate the quantity ratios in order to determine the full expression for the composite weights.

From the SNA we have the following six equations in the eight unknowns[6]

$p_{it}^{S}, p_{it+k}^{S}, p_{it}^{R}, p_{it+k}^{R}, q_{it}^{S}, q_{it+k}^{S}, q_{it}^{R}, q_{it+k}^{R}$

$$p_{it}^{S}q_{it}^{S} = V_{it}^{S} \tag{4}$$

$$p_{it+k}^{S}q_{it+k}^{S} = V_{it+k}^{S} \tag{5}$$

$$p_{it}^{R}q_{it}^{R} = V_{it}^{R} \tag{6}$$

$$p_{it+k}^{R}q_{it+k}^{R} = V_{it+k}^{R} \tag{7}$$

$$p_{it}^{S}q_{it+k}^{S} = K_{it+k}^{S(t)} \tag{8}$$

$$p_{it}^{R}q_{it+k}^{R} = K_{it+k}^{R(t)} \tag{9}$$

V_{it}^{S}, V_{it+k}^{S}, and $K_{it+k}^{S(t)}$ represent current and constant price values of the i'th component in S, and similarly for R.

All of the right hand sides of these equations are known.

We don't need to solve for these unkowns individually, however, just the ratios

$\frac{p_{it}^{S}}{p_{it}^{R}}, \frac{p_{it+k}^{S}}{p_{it+k}^{R}}, \frac{q_{it}^{S}}{q_{it}^{R}}, \frac{q_{it+k}^{S}}{q_{it+k}^{R}}$.

The problem then is to collapse equations (4) to (9) into four equations in these four unknown ratios.

(4) & (6) $$\Rightarrow \quad \frac{p_{it}^{S}}{p_{it}^{R}}\frac{q_{it}^{S}}{q_{it}^{R}} = \frac{V_{it}^{S}}{V_{it}^{R}} \tag{10}$$

[6]It is important to distinguish these SNA-based variables from their ECP-based counterparts, which we will do by placing a tilda over the ECP-based variables. Thus $\frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}}$ and $\frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}}$ will refer to component level PPPs from the ECP program.

(5) & (7) $$\Rightarrow \quad \frac{p^S_{it+k}}{p^R_{it+k}} \frac{q^S_{it+k}}{q^R_{it+k}} = \frac{V^S_{it+k}}{V^R_{it+k}} \qquad (11)$$

(8) & (9) $$\Rightarrow \quad \frac{p^S_{it}}{p^R_{it}} \frac{q^S_{it+k}}{q^R_{it+k}} = \frac{K^{S(t)}_{it+k}}{K^{R(t)}_{it+k}} \qquad (12)$$

While this makes only three equations in the four unknown ratios, we can set one of the ratios equal to its ECP-based counterpart.

Accordingly, let $$\frac{p^S_{it}}{p^R_{it}} = \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}} \qquad (13)$$

Solving equations (10) to (13) for the price and quantity ratios, and substituting into (3) we can express the ratio of implicit deflators, at the GDP-level, as:

$$\frac{\dfrac{\Sigma_i\, p^S_{it+k} q^S_{it+k}}{\Sigma_i\, p^S_{it} q^S_{it+k}}}{\dfrac{\Sigma_i\, p^R_{it+k} q^R_{it+k}}{\Sigma_i\, p^R_{it} q^R_{it+k}}} = \frac{\Sigma_i \left(W^R_{it+k} \left(\dfrac{p^S_{it+k}}{p^R_{it+k}} \right) \right)}{\Sigma_i \left(W^{R(t)}_{it+k} \left(\dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}} \right) \right)} \qquad (14)$$

where $W^R_{it+k} = w^R_{it+k} \dfrac{\tilde{p}^R_{it}}{\tilde{p}^S_{it}} \dfrac{K^{S(t)}_{it+k}}{K^{R(t)}_{it+k}}$ and $W^{R(t)}_{it+k} = w^{R(t)}_{it+k} \dfrac{\tilde{p}^R_{it}}{\tilde{p}^S_{it}} \dfrac{K^{S(t)}_{it+k}}{K^{R(t)}_{it+k}}$ which can be calculated from the available data.

C) Implications for Decomposition of real GDP growth rates

The differences between ECP-based and SNA-based real growth rates can be decomposed into a detailed set of different contributors with the help of (14). Recalling that the difference between ECP and SNA-based growth rates is given by expression (2), we can rewrite that expression, using (14), as:

$$\frac{\Sigma_i W^R_{it+k} \dfrac{p^S_{it+k}}{p^R_{it+k}}}{\Sigma_i W^{R(t)}_{it+k} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}} \; \frac{\left[\left(\Sigma_i\, \tilde{w}^S_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}} \right)^{1/2} \left(\Sigma_i\, \tilde{w}^R_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}} \right)^{1/2} + e^{SR}_{t+k} \right]^{-1}}{\left[\left(\Sigma_i\, \tilde{w}^S_{it} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}} \right)^{1/2} \left(\Sigma_i\, \tilde{w}^R_{it} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}} \right)^{1/2} + e^{SR}_{t} \right]^{-1}} \qquad (15)$$

Notice that we have started to systematically use the tilda notation to distinguish EC P-based entities from SNA-based entities. We have done this even for the weights to allow for the fact that there may have been some revision to the SNA between the time the ECP results were compiled (1996 or 2000), and when the SNA-based real growth rates were calculated.

If expression (15) is equal to 1, it indicates that there is no difference between the SNA-based and the ECP-based growth rates; greater than 1 indicates ECP-based growth is greater than SNA-based growth, and less than 1 the contrary. Expression (15) is the key to linking SNA-based and ECP-based entities.

For example, if we substitute ECP-based benchmarks $\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}$ into the numerator of the left-hand term we will remove from (15) any differences in the price information used in the two systems. Remaining differences will be due to differences in weights and in functional form, and we can sequentially remove these in a similar fashion, as will be seen below. We can use this strategy both broadly and in detail, to determine not only how much of the differences are due to differences in basic price data and different treatment of weights, but also how much each expenditure component contributes to differences in each of these categories.

a) Impact of differences in basic input data

For example, suppose we substitute $\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}$ for $\frac{p^S_{it+k}}{p^R_{it+k}}$ sequentially, starting with i = 1. The difference in (15) before and after the j'th substitution is made will tell us the impact that data differences $\frac{\tilde{p}^S_{jt+k}}{\tilde{p}^R_{jt+k}} - \frac{p^S_{jt+k}}{p^R_{jt+k}}$ make on the differences in aggregate real GDP growth rates. The sum over i of all such differences will tell us globally the impact of differences in basic data input on differences in real growth rates. ***This is the most fundamental measure of consistency between the SNA and ECP from the point of view of the consistency of the data building blocks.*** All subsequent differences are due to how the data are transformed within each of the two systems, or to revisions to the weights.

Once all substitutions have been made, (15) will look like

$$\frac{\dfrac{\sum_i W^R_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}}{\sum_i W^{R(t)}_{it+k} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}}}{\dfrac{\left[\left(\sum_i \tilde{w}^S_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i \tilde{w}^R_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2} + e^{SR}_{t+k}\right]^{-1}}{\left[\left(\sum_i \tilde{w}^S_{it} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i \tilde{w}^R_{it} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2} + e^{SR}_{t}\right]^{-1}}} \qquad (16)$$

with weights being the only remaining SNA entities.

We can express this in a form more similar to the PPP by writing:

$$\frac{\left(\sum_i W^R_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i W^R_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}}{\left(\sum_i W^{R(t)}_{it+k} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i W^{R(t)}_{it+k} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}} \; \frac{\left[\left(\sum_i \tilde{w}^S_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i \tilde{w}^R_{it+k} \dfrac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2} + e^{SR}_{t+k}\right]^{-1}}{\left[\left(\sum_i \tilde{w}^S_{it} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i \tilde{w}^R_{it} \dfrac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2} + e^{SR}_{t}\right]^{-1}} \qquad (17)$$

b) Impact of different treatment of weights

We can determine the impact of different treatment of weights in the SNA and the ECP by sequentially substituting ECP style weights for SNA-implied weights in the left hand term of (17). For example, in the numerator we replace W_{it+k}^{R} by w_{it+k}^{S} in one of the terms and by w_{it+k}^{R} in the other. Similarly, in the denominator replace $W_{it+k}^{R(t)}$ by w_{it}^{S} in one of the terms, and by w_{it}^{R} in the other. Notice that the weight replacements proposed for both the numerator and the denominator are without tildas, that is they reflect ECP style weights as calculated by the current SNA values, but consistent with the ECP Fisher formula. This will separate effects of different formulas in the two systems independently of any revisions to the SNA that might have been made between the current period and the time when the ECP results were compiled.

c) Impact of revisions to weights

One last round of substitutions, where original ECP weights $\tilde{w}_{it}^{S}, \tilde{w}_{it}^{R}, \tilde{w}_{it+k}^{S}, \tilde{w}_{it+k}^{R}$ are substituted for current SNA weights $w_{it}^{S}, w_{it}^{R}, w_{it+k}^{S}, w_{it+k}^{R}$, will capture the effect of SNA revisions on the differences between real GDP growth rates.

d) Impact of EKS adjustments

After making the weight substitutions outlined in c), (17) will read as:

$$\frac{1+e_t^{SR}\left(\sum_i \tilde{w}_{it}^{S}\frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}}\right)^{-1/2}\left(\sum_i \tilde{w}_{it}^{R}\frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}}\right)^{-1/2}}{1+e_{t+k}^{SR}\left(\sum_i \tilde{w}_{it+k}^{S}\frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}}\right)^{-1/2}\left(\sum_i \tilde{w}_{it+k}^{R}\frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}}\right)^{-1/2}} \qquad (18)$$

It is clear that if the EKS adjustment factors e_t^{SR} and e_{t+k}^{SR} are set to zero, (18) will equal 1. Therefore the deviation of (18) from 1 measures the effect of EKS adjustments on the difference between ECP and SNA-based real growth rates.

D) Accounting for Chaining

We have developed the analysis up to this point assuming that constant price entities were calculated in the prices of year t. In reality, however, in many countries constant price estimates are based on prices of the previous year. The derivation of (3) from (1) needs to be modified to take this into account.

We go back to equation (1) and modify it as before by multiplying and dividing by real growth in S between times t and t + k, and rearranging terms. The expression for real growth in S is different, however, when constant price entities are compiled in previous year's prices. In this case real growth in S from t to t + k will be the product of the growth rates in each of the intervening years, each calculated in prices of the previous year. In other words

$$\frac{KGDP_{t+k}^{S}}{KGDP_{t}^{S}}=\frac{\sum_i p_{it}^{S}q_{it+1}^{S}}{\sum_i p_{it}^{S}q_{it}^{S}}\frac{\sum_i p_{it+1}^{S}q_{it+2}^{S}}{\sum_i p_{it+1}^{S}q_{it+1}^{S}}\cdots\frac{\sum_i p_{it+k-1}^{S}q_{it+k}^{S}}{\sum_i p_{it+k-1}^{S}q_{it+k-1}^{S}}=\prod_{l=0}^{k-1}\frac{\sum_i p_{it+l}^{S}q_{it+l+1}^{S}}{\sum_i p_{it+l}^{S}q_{it+l}^{S}}$$

multiplying and dividing (1) by this expression, and rearranging terms gives:

$$\frac{\dfrac{\sum_i p_{it}^S q_{it+k}^S}{\sum_i p_{it}^S q_{it}^S}}{\prod_{l=0}^{k-1} \dfrac{\sum_i p_{it+l}^S q_{it+l+1}^S}{\sum_i p_{it+l}^S q_{it+l}^S}} \; \frac{\dfrac{\sum_i p_{it+k}^S q_{it+k}^S}{\sum_i p_{it}^S q_{it+k}^S}\left[\left(\sum_i w_{it+k}^S \dfrac{p_{it+k}^S}{p_{it+k}^R}\right)^{1/2}\left(\sum_i w_{it+k}^R \dfrac{p_{it+k}^S}{p_{it+k}^R}\right)^{1/2} + e_{t+k}^{SR}\right]^{-1}}{\dfrac{\sum_i p_{it+k}^R q_{it+k}^R}{\sum_i p_{it}^R q_{it+k}^R}\left[\left(\sum_i w_{it}^S \dfrac{p_{it}^S}{p_{it}^R}\right)^{1/2}\left(\sum_i w_{it}^R \dfrac{p_{it}^S}{p_{it}^R}\right)^{1/2} + e_t^{SR}\right]^{-1}}$$

instead of (2).

This is the same as (2) except it is pre-multiplied by the left hand term, the ratio of fixed base to chain estimates of growth in S. This means we can proceed as in the fixed base case, but remembering to multiply (3) by the ratio of fixed base to chain estimates of growth in S before using it to decompose contributions.

If GDP in the reference country R is also measured at previous year's prices, then a second correction factor, the ratio of fixed base to chain estimates of growth in R, must be introduced into (3). In the case the correction factor divides (3) rather then multiplying it.

E) Results

In this section we summarise the results obtained when the decomposition approach outlined above was applied to the preliminary ECP 2000 results shown in Chart 1. The empirical application required some preparatory work on the data, which is described first.

a) Data preparation

i) Redenomination

Several countries have redenominated their currencies since publication of the 1996 ECP results. While the ECE's macro-economic database for the SNA has incorporated these redenominations into the historical data record, the 1996 ECP results have not been adjusted in parallel, and it was necessary to do so. The following redenominations were made:

Austria	13.7603	Luxembourg	40.3399
Belguim	40.3399	Netherlands	2.20371
Finland	5.94573	Turkey	0.001
France	6.55957	Russia	1000
Germany	1.95583	Belarus	1000
Ireland	0.787564	Bulgaria	1000
Italy	1936.27		

ii) Expenditure Categories

The analysis required a set of expenditure categories that was simultaneously consistent across the 1996 ECP, the 2000 ECP, and the SNA. Not all countries in ECE's macro-economic database met this criterion. In order to encompass a reasonable number of countries that did share a common classification structure it was necessary to work at a high level of aggregation. As a result the analysis was carried out for a six-component breakdown of expenditure-based GDP. The components included in the analysis were:

Actual individual consumption
Collective consumption of general government
Gross Fixed Capital Formation
Changes in inventories & acquisitions less disposals of valuables
Exports of goods & services
Imports of goods & services
Gross Domestic Product

Imports and exports were treated separately in the SNA statistics, but together as the trade balance in the ECP statistics.

iii) Statistical discrepancy

One goal of the analysis is to determine the consistency of the basic expenditure price data underlying both systems. The SNA total expenditure on GDP often incorporates a statistical discrepancy, however, which reflects a compromise of the expenditure data with another balancing total such as GDP by activity, or income-based GDP. In order to have a pure comparison of one set of expenditure-based numbers with another, ECP data was compared to SNA total GDP excluding the stat istical discrepancy. Since the ECP program does not include a statistical discrepancy, this adjustment was also necessary to be consistent with the classification structure of the ECP.

For countries that report GDP in previous year's prices, the statist ical discrepancy was excluded by chaining growth in total GDP less discrepancy (in previous years prices) to its current price level in 1996

iv) Rebasing published fixed base estimates

In order to be consistent with the theoretical model outlined in earlier sections, countries that reported constant price GDP in prices of a fixed base year other than 1996 were rebased to 1996 at the six component level, and GDP excluding statistical discrepancy was calculated by summation of the components. In almost all cas es this involved rebasing form 1995 to 1996, and these adjustments were very small.

v) Estimating PPP for inventories

While the PPP for inventories was available for the ECP 1996 round, it was not available from preliminary data for the 2000 round, and it was necessary to estimate it residually by solving the equation

$$\frac{\tilde{p}_{GDPt}^{S}}{\tilde{p}_{GDPt}^{R}} = \left(\Sigma_i \, \tilde{w}_{it}^{S} \, \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} \left(\Sigma_i \, \tilde{w}_{it}^{R} \, \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} + e_t^{SR}$$

relating PPP at the total GDP level to its components. Assuming the EKS adjustment $e_t^{SR} = 0$ enables solving for the PPP for inventory change. B ecause of the assumption about the EKS term, however, the results will not be an exact replication of the PPPs that were actually used in the ECP 2000 calculations.

b) Decomposing differences in 1996-2000 growth rates

Table 1 shows the cumulative effects of substituting ECP-based PPPs and weights into expression (15). The substitutions were made sequentially: PPPs first (PPP1) then weights (weight2), and again with weights first (weight1) followed by PPPs (PPP2). The column labelled SNA shows the real growth rate as stated by the national accounts, while that labelled ECP shows the ECP-implied growth rate. The columns SNA and ECP are basically what is shown in Chart 1, with the exception that they exclude the effect of the statistical discrepancy.

We showed earlier that ECP = (15)*SNA, and that (15) converges towards 1 with progressive substitution of ECP-based entities for SNA-based entities. Each modification of (15) creates an associated implied SNA = ECP/(15) which would have been observed if the SNA had been based on the substituted ECP values. GDP in Belarus, for example, grew 41.1% according to the original SNA statistics. However, if we replace SNA price information with ECP price information, the growth rate falls to -14.7%, a massive narrowing of the difference between the original SNA and the ECP. If we then replace the SNA weights with ECP weights, growth falls a bit further to –17.6%. This suggests that most of the difference between ECP and SNA for Belarus is due to differences in the underlying price data, and not due to differences in weighting structures. The situation is somewhat different if we perform the weight substitutions first (weight1), and then the price substitutions (PPP2). The weight substitutions cause growth to fall about 20 points, from 41.1% to 21.9%, suggesting that weights have a larger influence than the previous scenario suggested, but still smaller than the price effect (PPP2), which reduces growth nearly 40 points more to –17.6%. The reason for the difference is interaction effects. If PPPs are substituted into a framework that still contains inconsistent treatment of weights (the PPP1/weight2 scenario), they will interact differently than they would with a consistent set of weights (the weight1/PPP2 scenario). This suggests that the importance of weights or of prices, excluding interactive effects, can be judged by the differences weight2-PPP1 and PPP2 – weight1 respectively.

After all price and weight substitutions have been made (15) does not converge completely to 1, and the implied SNA values do not converge completely to the ECP value. The residual, as indicated by expression (18), is due to the influence of the EKS adjustment at the overall GDP level[7]. This effect is indicated in the column labelled EKS/linking.

While individual rows Table 1 contain interesting information about the consistency of ECP and SNA estimates for each country, there is a need to summarise the general trend across all countries. This is done in the three bottom rows. The row labelled correlation shows the correlation co-efficient of ECP with each of the other columns. For example, the .34 under SNA indicates that the correlation of the original SNA figures with the ECP is quite poor. This rises to .90 and then to .96 with the PPP1 and weight2 substitutions respectively. On the other hand the weight1 PPP2 sequence shows correlation rises .19 points to .53 with consistent weights and then a further .43 points to .96 with the PPP substitutions. Recalling our earlier argument, the pure price effect, excluding interactions, is .43 points (.96-.53), while the pure weight effect is .06 (.96-.90). No matter how it is viewed it is clear that prices have a far larger role than weights in explaining the differences between ECP and SNA growth rates.

It must be kept in mind that we are working at the six-component level of GDP, and that prices at this level are themselves aggregates of lower level prices. It is possible that prices at this level in the two systems are different from each other because of weighting and aggregation below this level, and not because of differences in prices at the lowest level of disaggregation. While it seems implausible that weights should be a problem below a certain level but not above, it would nevertheless be reassuring to repeat the analysis presented here at a lower level of disaggregation.

It should also be noted that we were working with the final PPPs at the component level of the ECP. That means that the PPPs for each component contained the EKS adjustment, and were not a pure reflection of the data collection for that component. As a result, although we can say that most of the difference between ECP and SNA is due to prices, we do not know how much of the price difference is due to basic data collection and how much due to EKS adjustment. To answer this question would require that the analysis be redone using the non-EKS adjusted PPPs (the binary Fishers) to substitute into (15).

It is puzzling to observe that the mean EKS/Linking effect is not zero or thereabouts. Given the least squares nature of the EKS calculation one would expect a priori that the mean EKS adjustment would be near zero. However, the fixity requirement for Eurostat/OECD countries, combined with the linking of eastern European and CIS countries to the fixed Eurostat/OECD results, may be generating this effect, particularly since the link between the two groups was done with different countries in 2000 as compared to 1996. This suggests that one aspect of time-series continuity to keep in mind is to use the same link country from one round to the next.

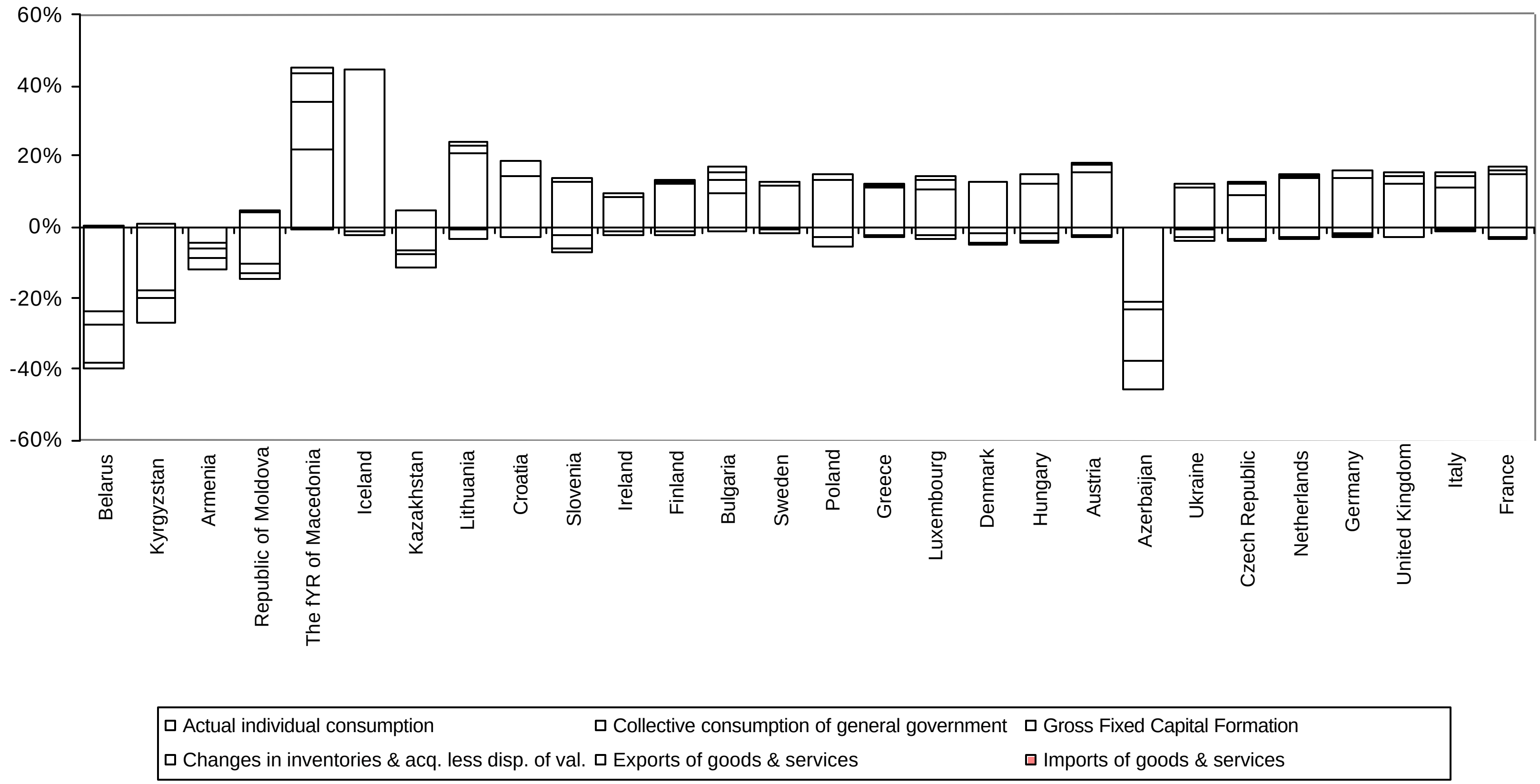
Knowing that price effects dominate the difference between ECP and SNA, it is natural to ask which prices are most responsible for the differences. Chart 2 shows the marginal contribution that substitution of PPPs associated with each expenditure component made to narrowing the SNA-ECP gap in the weight1-PPP2 scenario.

[7] This is not quite true. It would be if we were substituting non-EKS adjusted PPPs (binary Fisher indices) at the component level into (15). However, since the substitutions we made were the final PPPs, including EKS adjustment, the EKS/linking column indicates the combined (and possibly offsetting) effect of EKS adjustments at both the component and total GDP levels.

Table 1. Cumulative Effect of Price and Weight substitutions on 1996-2000 SNA growth rates

	ECP	PPP1	weight2	weight1	PPP2	SNA	EKS/ Linking
Belarus	-22.8	-14.7	-17.6	21.9	-17.6	41.1	-5.2
Kyrgyzstan	-30.4	-17.9	-33.5	-7.9	-33.5	19.7	3.1
Armenia	-6.9	3.9	-14.8	-2.6	-14.8	24.5	7.9
Republic of Moldova	-34.2	-17.5	-26.9	-16.9	-26.9	-6.1	-7.3
The fYR of Macedonia	44.4	83.0	42.9	-2.2	42.9	18.9	1.6
Iceland	2.5	9.0	10.7	-31.5	10.7	22.7	-8.2
Kazakhstan	-15.0	-2.8	-5.7	1.4	-5.7	4.1	-9.3
Lithuania	28.9	35.4	35.2	14.0	35.2	16.4	-6.3
Croatia	24.0	23.4	18.6	2.4	18.6	11.8	5.4
Slovenia	11.6	15.5	18.7	11.5	18.7	19.3	-7.1
Ireland	40.5	36.0	52.2	44.3	52.2	47.4	-11.7
Finland	18.9	21.5	29.5	18.2	29.5	24.5	-10.7
Bulgaria	10.4	21.2	22.0	6.3	22.0	5.3	-11.7
Sweden	11.7	15.4	20.1	8.4	20.1	16.0	-8.4
Poland	17.4	22.3	19.8	10.6	19.8	21.1	-2.3
Greece	11.9	20.7	18.9	9.2	18.9	15.6	-7.0
Luxembourg	37.9	25.0	54.5	43.0	54.5	34.6	-16.7
Denmark	8.7	11.8	15.8	7.6	15.8	11.4	-7.1
Hungary	18.6	25.4	22.8	12.5	22.8	21.0	-4.2
Austria	13.7	15.3	21.7	6.0	21.7	12.1	-8.0
Ukraine	6.2	11.0	2.4	-6.0	2.4	4.9	3.8
Czech Republic	3.3	2.7	2.9	-6.1	2.9	2.1	0.3
Netherlands	15.6	13.7	21.6	9.9	21.6	16.6	-6.1
Germany	9.6	14.5	15.1	1.4	15.1	8.7	-5.4
United Kingdom	12.3	19.7	22.3	9.5	22.3	12.8	-10.0
Italy	8.9	15.4	15.8	1.5	15.8	9.3	-6.9
France	12.5	18.3	20.9	7.4	20.9	12.6	-8.4
Correlation	1.00	0.90	0.96	0.53	0.96	0.34	
Mean	9.6	15.8	15.0	6.4	15.0	16.6	-5.4
Standard Deviation	19.0	19.2	21.1	15.3	21.1	11.5	5.8

Chart 2. Marginal Contribution of component level price substitutions on the 1996-2000 SNA growth rates



The effect is called “marginal” in that it measures the cumulative effect of substituting PPPs up to the i’th expenditure component less the cumulative effect of substituting up to the i-1’st expenditure component. The chart shows that the largest and most widespread effect is due pricing differences associated with Actual Individual Consumption. It is very sizeable in virtually every country and over 20 percentage points in absolute value in 4 countries. No other expenditure component is uniformly important in all countries. This partly reflects the large weight of Actual Individual Consumption in GDP, but nevertheless points to this area is the most important source of potential data improvements to narrow the gap.

Pricing of other expenditure components plays a very sizeable role only in certain countries. Gross fixed capital formation is the second most significant contributor, and has a particularly prominent impact in Belarus, Kyrgystan, Macedonia, and Azerbaijan, and a more moderate, but still substantial role, in several other countries. Collective consumption of general government is quite important in explaining the differences for Macedonia, but more moderate elsewhere. The PPP for inventories is extremely large for Iceland, but recall that the PPPs for inventories had to be estimated residually for the ECP 2000 round. With actual PPPs used for inventories this effect would likely shrink to a small size.

It is striking that the trade balance does not figure more prominently in reconciling the differences between the two systems, with Kazakhstan being the only country to exhibit a substantial impact due to this component. While this may be correct, it also points to the need for further reflection and a better understanding of the approach proposed in this paper, especially of the way trade is handled in this analysis [8].

As a general comment it needs to be stressed that identification of inconsistencies in the basic input price data in the two systems does not point to one or the other as the weak link. It does, however, narrow the field of focus for further investigation. As Magnien (2002) pointed out, other comparisons, over time and across countries, should be able to help point to whether SNA or ECP data are in need of improvement.

F) Conclusion

We have pointed out the need for a framework for making a comprehensive and exact reconciliation between SNA-based real growth and ECP-implied real growth. A theoretical framework for such a reconciliation has been developed, and outlined in this paper. It has been tested by applying it to the preliminary results for the ECP 2000 round. While these tests have provided useful information about the SNA/ECP differences, they should at this stage be regarded more as illustrative than conclusive. They have been based on an incomplete set of data: the range of countries was limited because data with a common classification structure was not available; the ECP 2000 PPP for inventory change had to be estimated residually; the lack of access to binary Fisher PPPs prevented us from determining to what extent differences in basic input data was due to data collection as opposed to EKS adjustments at the six-component level; and finally, having carried out the analysis at the six-component level, it is uncertain whether differences in basic input price data identified at this level are due to weighting effects in aggregation from lower level PPPs, or whether they are due to differences in data collection at the lowest level. Virtually all of these caveats can be overcome with the use of a more complete dataset. The approach also gave some puzzling results such as little sensitivity to PPPs for the external trade balance, and a non zero cross country average for the impact of the GDP level EKS adjustment. This points to the need for further reflection and understanding of the basic approach before it can be used to draw more definite conclusions about the ECP/SNA differences.

[8] The SNA provides implicit deflators and shares for both exports and imports, while in the ECP there is only one PPP, and a single share for the trade balance. The scheme for integrating these two frameworks for the sensitivity analysis carried out for this paper may be worth reviewing.

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