# Data Evaluation with Consecutive Censuses: Census Coverage 

United Nations Statistics Division

## Outline

## 1. The Population Balancing Equation

2. Intercensal Cohort Survival Rates
3. Cohort Component Method

## Population Balancing Equation

## Census coverage

## Population balancing equation

## If a country has:

$\square$ A relatively complete system of vital registration
$\square$ A fairly reliable estimate of the degree of under-registration
> Information on the number of intercensal births, deaths and net international migrants can be used in conjunction with the results of a previous census to evaluate the coverage of a subsequent or current census.

## Population balancing equation

$$
P_{1}=P_{0}+B-D+M
$$

Where:
$P_{1}=$ the population enumerated in the census being evaluated
$P_{0}=$ The population enumerated in a previous census
$B=$ the number of births in the period between the two censuses
$D=$ the number of deaths in the period between two censuses
$M=$ the number of net international migrants in the period

$$
\mathrm{M}=\mathrm{I} \text { (Immigrants) }-\mathrm{E} \text { (Emigrants) }
$$

## Population balancing equation

$\square$ The population balancing equation is the most fundamental equation in demographic analysis and is also used to estimate population growth.
$\square$ It is based on the logic that:
The population of a country can increase or decrease between any two points in time only as a result of births, deaths and movement of population across national boundaries
> Births and immigration add to the population
> Deaths and emigration reduce the population

## Population balancing equation

$\square$ For census evaluation purposes, there is a residual (e) needed to make the equation balance exactly
$\square$ " $e$ " in the equation is referred to as the "error of closure" and represents the balance of errors in the data on births, deaths, net migration, and the coverage of the two censuses:

$$
P_{1}=P_{0}+B-D+M+e
$$

$>$ If a negative residual quantity $e, P_{1}$ is under-enumerated relative to $P_{0}$
$>$ If a positive residual is required to balance the equation, $P_{1}$ is over-enumerated relative to $P_{0}$

## Population balancing equation - Data required

$\square$ The population enumerated in two consecutive censuses
$>P_{1}$ : the census under evaluation
$>P_{0}$ : previous census
$\square$ The number of births, deaths and net international migration (immigrants-emigrants) during the intercensal period, adjusted for under-registration (to the extent possible)

## Population balancing equation Computational Procedure

1. Compile registered numbers of intercensal births, deaths and migrants

- Vital registration system
- Immigration record system (residence permit, border records, etc.)
> Adjustment based on under-coverage of these systems including indirect estimates

2. Calculation of the "expected" census population ( $\mathrm{E}(\mathrm{P} 1)$ )

$$
E(P 1)=P 0+B-D+M
$$

3. Calculation of the residual error or error of closure

$$
e=P_{1}-E\left(P_{1}\right)
$$

## Population balancing equation Interpretation of "e"

If $P_{0}$ has been adjusted for net coverage error, the estimated residual error (e) will represent an estimate of net coverage error in $\mathrm{P}_{1}$
$>$ If " $e$ " is positive, $P_{1}$ is over-enumerated
$>$ If " $e$ " is negative, $\mathrm{P}_{1}$ is under-enumerated

If $P_{0}$ is not adjusted, " $e$ " will represent an estimate of the relative level of net coverage error in $\mathrm{P}_{1}$ in comparison with $\mathrm{P}_{0}$

Population balancing equation - Example
Sri Lanka, 1971 and 1981 Censuses (1)

## For an unadjusted census:

$$
\begin{aligned}
& \begin{aligned}
\mathrm{E}\left(\mathrm{P}_{1}\right) & =\mathrm{P}_{0}(\text { unadjusted })+\mathrm{B}_{\text {adj }}-\mathrm{D}_{\text {adj }}+\mathrm{M}_{\text {adj }} \\
& =12,689,897+3,716,878-1,002,108+(-446,911) \\
& =14,957,756
\end{aligned} \\
& \begin{aligned}
e & =P_{1}-\mathrm{E}\left(\mathrm{P}_{1}\right) \\
= & 14,848,364-14,957,756 \\
= & -109,3920.7 \% \text { of } E\left(P_{1}\right)
\end{aligned} \\
& P_{1} \text { is under-enumerated relative to } P_{0}
\end{aligned}
$$

[^0]Population balancing equation - Example
Sri Lanka, 1971 and 1981 Censuses (2)

## For an adjusted census count:

$$
\begin{aligned}
\mathrm{E} & \left(\mathrm{P}_{1}\right)=\mathrm{P} 0(\text { adjusted })+\mathrm{B}_{\text {adj }}-\mathrm{D}_{\text {adj }}+\mathrm{M}_{\text {adj }} \\
& =12,849,796+3,716,878-1,002,108+(-446,911) \\
& =15,117,655 \\
e & =P_{1}-E\left(P_{1}\right)=14,848,364-15,117,655 \\
& =-269,291 \text { 1.8\% of } E\left(P_{1}\right) \\
& P_{1} \text { is under-enumerated }
\end{aligned}
$$

[^1]
## Population balancing equation - Limitations

$\square$ Incomplete and defective data on the components of population change are very common

- Applicability of the method is limited to countries with good vital registration coverage and migration data
$\square$ It is generally not useful for obtaining estimates of net census coverage error for sub-national populations (for example regions, provinces).
- In addition to the components of population change considered, internal migration has to be considered.
> For most practical purposes, the use of the population balancing equation is limited to analysis of net coverage error at the national level.


## Cohort Survival Ratios

Mortality and census coverage

## Cohort survival ratios

$\square$ This technique is based on a comparison of the size of birth cohorts enumerated in successive censuses
$\square$ In the absence of census errors, the ratio of the number of persons in a cohort enumerated in the second census to the number enumerated in the first census should approximate the survival rate that would be expected on the basis of mortality conditions

- E.g. we have a cohort of males aged 40-44 at the time of the first census, say in 2000
- If the next census is held exactly 10 years later, in 2010, this cohort will be aged $50-54$
- In the absence of other factors, we expect their numbers to have been reduced only by the life table quantity ${ }_{10} d_{x}=I_{x}-I_{x+n}$, the number of deaths to those aged $x$ over the subsequent 10 years


## Cohort Survival Ratios: Caveats

The method is less useful when other factors make it difficult to determine whether deviations from the expected CSR are due to census error or something else

- Substantial net migration (unless there are accurate estimates of net migration by age)
- Changes in country borders between censuses
- Changes in the population groups included in the two censuses (e.g. active military, nomadic groups) if the size of these groups is substantial


## Cohort Survival Ratios: Jordan, 1994 \& 2004 Censuses



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## Cohort Survival Ratios: DPR Korea



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## Calculating CSRs (1)

Intercensal cohort survival rates are defined as:

$$
{ }_{n} \operatorname{CSR}_{x}(a)=\frac{{ }_{n} P_{x+a}(t+a)}{{ }_{n} P_{x}(t)}
$$

Where:

- $t=$ time of first census
- $a=$ number of years between censuses
- ${ }_{n} P_{x}(t)=$ size of the cohort at the time of the first census
- ${ }_{n} P_{x+a}(t+a)=$ size of the cohort at the time of the second census


## Calculating CSRs (2)

The ratio of the observed intercensal cohort survival rate to the corresponding life-table survival rate

$$
{ }_{n} P_{x+a}(t+a) /{ }_{n} P_{x}(t)
$$

${ }_{n} R_{x}=$

$$
{ }_{n} L_{x+a} /{ }_{n} L_{x}
$$

## Where:

- ${ }_{n} P_{x+a}(t+a)=$ size of the cohort at time of the second census
- ${ }_{\mathrm{n}} \mathrm{P}_{\mathrm{x}}(t)=$ size of the cohort at the time of the first census
- ${ }_{n} L_{x+a}=$ life table number of person-years lived in age interval $x+a$ to $x+a+n$ years
- ${ }_{n} L_{x}=$ life table number of person-years lived in age interval $x$ to $x+n$ years


## Cohort Survival Ratio - Interpretation

In the absence of census error, the expected value of the ratio $\left({ }_{n} R_{x}\right)$ would be 1.0

Ratio values for any particular cohort which exceed 1.0 would indicate over-enumeration of the cohort in the second census relative to the first census

Ratio values of less than 1.0 would indicate underenumeration of the cohort in the second census relative the first census

## Cohort survival ratios - Example (1)

Step 1: Adjustment for migration (if appropriate)
$\square$ In countries experiencing significant levels of net intercensal immigration, the number of net immigrants in each cohort may either added to the cohort enumerated in the first census or subtracted from the cohort enumerated in the second census
$\square$ In cohorts experiencing net intercensal emigration, the number of net intercensal emigrants can either added to the second census or subtracted from the first census
$\square$ Should be confident that migration data is reasonably accurate before making any adjustments

## Cohort survival ratios - Example (2)

Step 2: Calculation of census survival rates using two consecutive censuses ${ }_{n} P_{x+a}(t+a) /{ }_{n} P_{x}(t)$


## Cohort survival ratios - Example (3)

Step 3: Calculation of life table survival rates based on the expected level of mortality

$$
n S x=(n L x+a / n L x)
$$

Step 4: Calculation of cohort survival ratios ( nRx )


## Cohort survival ratios - Example (4)

Census and life table survival rates for Brazilian females 2000-2010


## Cohort Survival: Uses and limitations

$\square$ It is a widely applicable approach for examining error in consecutive censuses
$\square$ Method requires relatively little information

- Information on the level of fertility is not required since the method does not assess the coverage of the population born between two censuses
$\square$ Method is complicated by migration etc. as discussed
- When only two censuses are available, the method suffers from the limitations shared by many demographic methods, namely difficulties in separating census errors from real irregularities caused by extraordinary events
- The utility of census survival approaches increases significantly when three or more censuses are available


## Cohort Component Method

Census coverage

## Overview of cohort-component method



Smith, S., J. Tayman, and D. A. Swanson. 2001. State and Local Population Projections: Methodology and Analysis. New York: Kluwer Academic/Plenum Publishers.

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## Cohort component method

In this approach, the population enumerated in the first census is projected to the reference date of the second census based on estimated levels and age schedules of fertility, mortality and migration during the intercensal period
$\square$ The expected population from the projection is then compared with the actual population enumerated in the second census

- Data for intercensal births, deaths and migration are taken from estimates and/or assumptions regarding the level and age schedules of these parameters rather than directly available data based on registration systems


## Cohort component method - data required

1. The population enumerated in two censuses by age and sex
2. Age specific fertility rates for women aged 15 to 49 (in 5 -year age groups), assumed to represent the level and age structure of fertility during the intercensal period
3. Life table survival rates for males and females, assumed to be representative of mortality conditions during the intercensal period
4. An estimate of sex ratio at birth
5. Estimates of the level and age pattern of net international migration during the intercensal period if the level of net migration is substantial

## Cohort component method overview of computational steps

1. "Survive" the age distribution at the initial census to the time of the second census
2. Multiply each age group population by life table survival rates
3. Open-ended interval requires special handling
4. Make any necessary adjustments for migration
5. Calculate the number of births during the period
6. Average initial and projected population for each age group between $15-49$ to estimate mid-period female population
7. Apply age-specific birth rates to these populations to generate total numbers of births during time period
8. Apply sex ratio factor to get female and male births from total births
9. Apply life table survivorship to these births to determine number that survive to time of the second census
10. Compare the estimated female population by age group with the enumerated female population

## Cohort component method -

## Step 1 (survive initial age distribution)

1. "Survive" the age distribution at the initial census to the time of the second census



## Cohort component method Step 1 (survive initial age distribution)

2. For the oldest age category (open-ended)

$$
{ }_{w} S_{x}=\frac{{ }_{w} T_{x+a}}{{ }_{w} T_{x}}
$$

$w=$ the oldest age attainable in the population
$a=$ the length of the projection interval
${ }_{w} S_{x}=$ the life table survival ratio for the
population aged $x$ and above
${ }_{k} T_{x}=$ the number of life table persons lived at ages $x$ and above
$r^{T_{x+a}}=$ the number of life table person-years
lived at ages $x+a$ and above


## Cohort component method Step 2 (adjust for migration)

If net international migration is substantial, the "survived" cohort population must be adjusted to reflect the effects of migration
$>$ The introduction of net migrants by age group at the mid-point of the projection period and the survival of net migrants to the end of the period:

$$
{ }_{n} \hat{M}_{x+i}=\frac{1}{4}{ }_{n} M_{x}\left(1+{ }_{n} S_{x}\right)+\frac{1}{4}{ }_{n} M_{x+i}\left(1+{ }_{n} S_{x+i}\right)
$$

Assumptions: i) An equal distribution of net migrants across years of the intercensal period, ii) Migrants have the same fertility and mortality level as the enumerated population

## Cohort component method Step 3 (calculate births)

1. Calculate the average number of women in each childbearing age group (15-49) during the intercensal period in order to estimate the number of births during the projection period

$$
{ }_{n} \bar{P}_{x}=\frac{{ }_{n} P^{0}{ }_{x}+{ }_{n} P^{1}{ }_{x}}{2}
$$

${ }_{n} P_{x}=$ average number of females aged $x$ to $x+n$ in the projection period
${ }_{n} P^{0}{ }_{x}=$ number of females aged $x$ to $x+n$ at the beginning of the projection period
${ }_{n} \overline{P^{l}}{ }_{x}=$ projected number of females aged $x$ to $x+n$ at the end of the projection period

## Cohort component method Step 3 (calculate births)

2. Calculate total births during the period
$B=\sum_{x=15}^{49}\left({ }_{n} \bar{P}_{x} *{ }_{n} f_{x}\right)$ for 1 -year projection
$B=5 * \sum_{x=15}^{49}\left({ }_{n} \bar{P}_{x} *{ }_{n} f_{x}\right)$ for 5 -year projection period
$B=$ the estimated number of births during the projection period
${ }_{n} \overline{\mathrm{P}}_{\mathrm{x}}=$ the average number of women in the age group x to $\mathrm{x}+\mathrm{n}$ years during the projection period
${ }_{\mathrm{n}} \mathrm{f}_{\mathrm{x}}=$ the age specific fertility rate (per woman) for women age x to $x+n$ years during the projection period


## Cohort component method -

 Step 3 (calculate births)3. Calculate proportion of male and female births

$$
\begin{aligned}
& \text { SRB } \\
& 1+\text { SRB } \\
& \text { SRB } \\
& \mathrm{B}^{\mathrm{m}}=\quad \square \\
& \text { 1+ SRB } \\
& \mathrm{B}^{\mathrm{f}}=1-(1.05) /(1+1.05)=.488 \\
& =.488 * 17,521,248=8,550,369 \\
& \mathrm{~B}^{\mathrm{m}}=(1.05) /(1+1.05)=.512 \\
& =.512 * 17,521,248=8,970,879
\end{aligned}
$$

## Cohort component method -

## Step 4 (survive intercensal births)

3. Apply life table survivorship to these births to determine number that survive to time of the second census

$$
\begin{aligned}
& { }_{5} \mathrm{P}_{0}^{\mathrm{f}}=\mathrm{B}^{\mathrm{f} *}{ }_{5} \mathrm{~S}_{0} \\
& { }_{5} \mathrm{~S}_{0}=\frac{{ }_{5} \mathrm{~L}_{0}}{5 * \mathrm{l}_{0}}
\end{aligned}
$$

$$
{ }_{5} \mathrm{~S}_{0}=484,129 / 500,000=.968
$$

$$
{ }_{5} \mathrm{P}_{0}^{\mathrm{f}}=.968 * 8,550,369=8,276,757
$$

## Cohort component method -

## Step 5 (compare with enumerated population)

Final step in procedure is to compare the enumerated population by age and sex in the second population with the expected population


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## Cohort component method in MortPak (1)



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## Cohort component method in MortPak (2)



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## Brazil Example with MortPak - Results



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## Brazil Example with MortPak - Results



## Main findings from Brazil example

$\square$ Suggests underenumeration by about 1.5 million people, or 0. $8 \%$ of the population

- Significant underenumeration of youngest two age groups, particularly children 0-4
$\square$ Some overenumeration of 10-14 year olds - could be a result of the underenumeration of this group (as $0-4$ year olds) in the original census in 2000
$\square$ Seeing same under-enumeration of 25-29 year olds of both sexes as when we calculated by hand - needs to be explored
$\square$ Consistent but fairly low level of overenumeration of adults age 35-39 to 75-79
- Could potentially indicate in-migration, ideally want to incorporate migration data
$\square$ Significant overenumeration of older people
- Very likely that there is age exaggeration
- Also might consider that our life table is not accurate for these ages


## Kenya (1) - 2009 census



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## Kenya (2)



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## Kenya (3)

$\square$ Overall suggests net undercount of $1.1 \%$, about 460,000 people
$\square$ Most of undercount is coming from males aged 20-45 and females aged 25-59

- Migration may account for some of this difference
- The life table used (based on Kenya 1999) census may not accurately represent changing mortality conditions over the 10 year period due to the HIV/AIDS epidemic
- There may be a 'true' undercount of these age-sex groups


## Cohort component method uses and limitations

$\square$ It is applicable when registration data are not-existent or deficient to such an extent that satisfactory adjustment is not possible
$\square$ Sufficient information to derive estimates of fertility and mortality levels should be available

- Mortality estimates can be complicated by HIV/AIDS - with a generalized epidemic, one life table is generally not sufficient to model mortality patterns over a 10 year period
$\square$ Lack of information on international migration is often a problematic issue when applying this method
$\square$ In case where sufficient information exists to derive reliable estimates of demographic parameters, the method is perhaps the most powerful among the alternative demographic approaches for the evaluation of censuses, since it provides age and sex specific estimates of net census error


## Tools



PROJCT application in MORTPAK - The United Nations software package for demographic measurement, available online: http://www.un.org/en/development/desa/population/publications/mortality/ mortpak.shtml

DemProj module in Spectrum, available online at:
http://www.futuresinstitute.org/spectrum.aspx

- DemProj is recommended for projection in contexts in which HIV/AIDS prevalence exceeds a few percent - better modeling of mortality conditions
- Requires more data input, including prevalence and treatment estimates for HIV/AIDS
- Data input options somewhat less flexible than MortPak


## References

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[^2]
[^0]:    Source: U.S. Census Bureau (1985)

[^1]:    Source: U.S. Census Bureau (1985)

[^2]:    United Nations Workshop on Revision 3 of Principles and Recommendations for Population and Housing Census and Census Evaluation Amman, Jordan

