Workshop on the Methodological Review of Benchmarking, Rebasing and Chain-linking of Economic Indicators

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A review of the use of price index in national accounts

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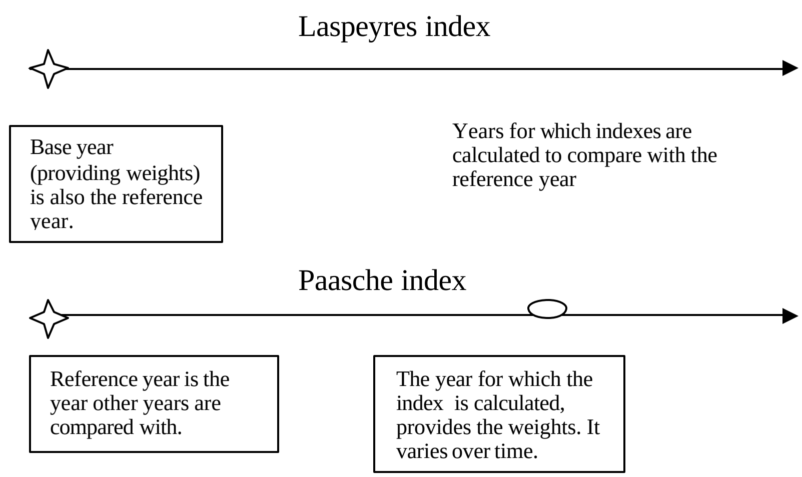
This review aims at further clarifying chapter XVI, "Price and Volume Measures", of the 1993 SNA with respect to the impact on rates of changes and data requirement when different types of indexes are used in national accounts. Indexes are a particularly difficult issue to study when they are applied to track changes in volume or price of a group of goods and services. A group of goods and services, for example food, which is a bundle of diverse commodities, is an abstract concept with no explicit quantity or price. To study it over time, statisticians create volume and price indexes. Until now, no perfect solution has been obtained, but some solutions are better than others. The paper tries to explain advantages and disadvantages of the solutions that are commonly used. Part I of the paper discusses the Laspeyres, Paasche and Fisher indexes as direct indexes when the base year is fixed. Part II discusses the linking of different series with different base year and the solution to avoid the rewriting of history (rates of changes in prices and volume change) created by the shifting of the base year periodically. Part III discusses specific issues of national accounts.

I. Direct volume and price index

A direct index (volume or price) is calculated to compare two points in time. It is essentially a binary index where an index of a given point is calculated in comparison to another point. This type of binary index disregards the information of the intervening periods between the two points. The Laspeyres index uses a previous period to serve as the *base*, i.e. it uses their prices or quantities as weights. To each Laspeyres index, a correspondent index is attached, which is called Paasche index; it uses prices and quantities of the latter period as weights or as the base period. The Fisher index is the geometric mean of these two indexes.

Figure 1: Illustration of Laspeyres and Paasche index



The indexes, however, are not calculated simply to compare any pair of years only. Analysts want indexes to depict real volume and price changes over time. As a consequence, for a time series, a *reference year* must be selected so that other years can be compared with it to derive the indexes. For direct indexes, specifically the Laspeyres, the calculation is still done in a binary way, i.e. an index for any given year is calculated to compare it with the year whose prices and quantities are used as weights. In this case, the year that provides the weights and the point of reference is called the *base year.* Individual indexes are still calculated without taking into account data in the intervening years.

In a time series, corresponding to each of this Laspeyres index is a Paasche index. So even though prices and quantities of the current year are used as weights in a Paasche index, implicitly the base for the comparison is still the *base year* of the associated Laspeyres index. A direct Fisher index as the geometric mean of the other two indexes therefore must be based on the same initial base year.[1]

Given a series of indexes with a base year[2], that base year is initially also used as the reference year, but later for convenience, another *Reference year* can be selected for the series. This is the year selected to have the index of 100. Indexes of other years can be

[1] The issue will be clear when table 6 is explained.
[2] A year that is selected as the base year should be a normal year which is not in economic crisis so that the economic structure of that year is not atypical.

mechanically grossed up or down proportionally to the original series of indexes, but its *base year* does not change.

Due to the close interrelationships between the Laspeyres, Paasche and Fisher indexes, when a new base year is selected for the Laspeyres indexes, the corresponding Paasche and Fisher indexes also change. These direct indexes will be explored in detail below.

1. Laspeyres index

The Laspeyres volume index uses prices of a given base year as fixed weights. The volume index of year t showing the growth from year 0 and year t, uses prices at year 0 as weights. The price index uses quantities of year 0 as weights. Formulas are as follow where the sum is over all elements in the group and the base weights are in parenthesis, q stands for quantity and p stands for price:

(1) $L_{qt}(p_0)= \sum p_0q_t / \sum p_0q_0$ (Laspeyres volume index)

(2) $L_{pt}(q_0)= \sum p_tq_0 / \sum p_0q_0$ (Laspeyres price index)

2. Paasche index

The Paasche volume index uses prices of the current year t as fixed weights. The volume index of year t showing the growth from year 0 and year t uses prices at year t as weights. The price index uses quantities of year t as weights. Formulas are as follow where the sum is over all elements in the group and the base weights are in parenthesis:

(3) $P_{qt}(p_t)= \sum p_tq_t / \sum p_tq_0$ (Paasche volume index)

(4) $P_{pt}(q_1)= \sum p_tq_t / \sum p_0q_t$ (Paasche price index)

3. Relationship between Laspeyres and Paasche indexes

Indexes are used to measure volume changes (i.e. change in value at constant prices) and price changes over time. Volume in constant prices can be obtained by multiplying the base year value by the quantity index or by deflating the value in current prices. The relationships between Laspeyres and Paasche indexes can be better seen in this light.

a) *Laspeyres value at constant prices of the base year is obtainable by deflating current value with its associated Paasche price index.*[3]

[3] The value of a group of products at time t at constant prices at base year 0 is the sum of quantities at time t multiplied by their prices at the base year 0:

$$V_{t,co} = \sum p_0q_t$$

b) *Laspeyes value at constant prices is also obtainable by extrapolating the base year value with its Laspeyres volume index.*[4]

c) *For any given year, the Laspyres volume index * Paasche price index = Paasche volume index * Laspeyres price index.*

d) *The most commonly used types of indexes in statistics until recently are the Laspeyres volume indexes and their implicit Paasche price indexes (property a). A common practice has been as follows: first, Laspeyres volume indexes are computed; second, values at constant prices are then derived by extrapolation (property of b); third, Paasche price indexes (implicit) are indirectly derived by dividing values at current prices with values at constant prices (property a).*

These relationships are shown or can be checked in the example given in table 1, which is calculated from raw data in table A in the appendix.

Table 1. Relationship between Laspeyres and Paasche indexes

	Year 20 compared to year 0	
	Volume index	Price index
Base year 0 (Laspyeres)	3.230769	1.06153846
Base year 20 (Paasche)	2.521739	0.82857143
Current value of year 0	65	
Current value of year 20	174	
Calculating constant value and price index of year 20, base year = year 0		
Constant value of year 20 by extrapolating	Current value of base year*Laspeyres volume index = 65*3.230769 = 210	
Price index (Paasche)	Current value/constant value = 210/174 = 0.82857143	

Data sources: see table A, appendix.

Multiplying and dividing the right-hand side by the same current value at time t ($V_t = \sum p_t q_t$) will not change the value of the left-hand side. The result is shown below. Then, the Laspyeres constant value equals the current value deflated by the Paasche price index $P_{pt} = (\sum p_t q_t / \sum p_0 q_t)$.

$$V_{t,co} = \sum p_0 q_t = \sum p_t q_t / (\sum p_t q_t / \sum p_0 q_t)$$
$$= V_t / P_{pt}$$

[4] The value at constant value at time t ($V_{t,co} = \sum p_0 q_t$) can be rewritten by multiplying and dividing the right-hand side by the same value at the base year $V_0 = \sum p_0 q_0$:

$$V_{t,co} = \sum p_0 q_t = \sum p_0 q_0 * (\sum p_0 q_t / \sum p_0 q_0) = V_0 * L_{qt}$$

4. Fisher index

The Fisher ideal indexes are the geometric means of the Laspeyres and Paasche indexes:

$$(5) \qquad F_{qt} = (L_{qt})^{1/2} * (P_{qt})^{1/2} \qquad \text{(Fisher volume index)}$$
$$(6) \qquad F_{pt} = (L_{pt})^{1/2} * (P_{pt})^{1/2} \qquad \text{(Fisher price index)}$$

The Fisher indexes are called ideal indexes as they satisfy the following tests:

- *Time reversal*: the time reversal requires that the index for period t based on 0 should be the reciprocal of that for 0 based on t;
- *Factor reversal*: the factor reversal requires that the product of the price index and volume index should be equal to the proportional change in the current values.

Laspeyres and Paasche indexes do not satisfy either of the tests. (The time reversal is clearly violated because the Paasche index is the reverse of the Laspeyres index but the values are not the same; the violation of the factor reversal is reflected in the fact that one cannot obtain constant value by deflating current value using the price index of the same type). Besides that, the Laspeyres and Paasche indexes do not reflect accurately the rates of growth in volume when the periods under examination are far away from the base year.

Different types of volume indexes and resulting volume growth rates are calculated and shown in table 2. Data used for the calculation are taken from table A in the appendix.

Table 2. Volume indexes and growth rates by different types of indexes

	Year 0,0	Year 0,10	Year 0,15	Year 0,20
Laspeyres				
Index	1	1.2923	2.6154	3.2308
Growth rate over previous period		29.23%	102.38%	23.53%
Paasche				
Index	1	1.2938	2.3200	2.5217
Growth rate over previous period		29.38%	79.32%	8.70%
Fisher				
Index	1	1.2930	2.4633	2.8543
Growth rate over previous period		29.30%	90.50%	15.87%

Data sources: see table A, appendix.

The indexes in table 2 are from year 0 to year 10, year 15 and year 20 respectively. Growth rates over previous years are calculated using those indexes. The following observations can be drawn from table 2:

a) *Laspeyres volume indexes are mostly higher than those of Paashce volume indexes*. This is true only if there is a substitution effect in the economy, i.e. a decrease in the relative price of a product shifts expenditures to another product. This is an important rule in economic theory. In our example (table A in the appendix), we assume that there are high-tech and non high-tech goods, it happens that prices of high-tech goods decline relative to those of non high-tech goods and a higher share of expenditures is diverted to high-tech goods, except for year 10. This assumption is reasonable, as lower relative prices do not necessarily lead to substitution because either substitution takes time to effect or technical requirement does not allow substitution. For this reason, the Laspeyres volume index in year 10 is lower than the Paasche index.

b) *Fisher volume indexes are always between those of Laspeyes and Paasche volume indexes.* Thus as compared to Fisher indexes, Laspeyres indexes tend to provide higher volume growth rates for the current year and years close to the current year and Paasche indexes tend to provide lower growth rates for the current year, taking into account the clarification in (a).

c) *Paasche price indexes tend to overshoot the changes in prices*. This observation is important in political and economic decision-making, as wages and transfer incomes tend to be adjusted automatically in many countries by changes in consumer price indexes.

d) *With Fisher indexes, one can obtain the same value at constant prices by using the volume index to extrapolate a value at the base year or by dividing current value by its Fisher price index*. This is not possible with Laspeyres or Paasche indexes.

Table 3. Comparison between Fisher index and Laspyeres index: volume in base year prices
(Base year: year 0)

	Year 0	Year 10	Year 15	Year 20
Laspeyres				
1. Non high-tech	45	60	90	90
2. High-tech	20	24	80	120
3. Total (1+2)	65	84	170	210
4. Volume index	1.0	1.2923	2.6154	3.2308
5. Volume total extrapolated by index[5]	65	84	170	210
Fisher				
1. Non high-tech	45	60	90	90
2. High-tech	20	24	80	120
3. Total (1+2)	65	84	170	210
4. Volume index	1.0	1.2930	2.4633	2.8543
5. Volume total extrapolated by index	65	84.05	160.1	185.5

Data sources: see table A, appendix.

Additivity problem: For the above-mentioned crucial reasons in section 4, the SNA has recommended the use of Fisher indexes. However, the Fisher index has one problem that is not faced by the Laspeyres and Paasche indexes, i.e. *the total volume derived by the volume Fisher index is not equal to the sum of the components at constant price. The difference is magnified by the distance between the current year and the base year*. For years 15 and 20, volumes derived by the Fisher indexes through extrapolation, 160.1 and

[5] Any difference in the total of the components and the total derived by using the indexes to extrapolate the base year value is due to rounding error.

185.5, are quite different from 170 and 210 respectively (see table 3). Laspeyres and similarly Paasche give the same totals.

II. Problems when base year changes

1. Effects of change in base year

As the base for an index series moves farther from the current year, the structure in the components changes. For example in our example in table A in the appendix, more high-tech goods are used as their prices decline. Laspeyres volume indexes, which use prices of the base year as weights, give higher value to the nominator $\Sigma p_0 q_t$ in the Laspeyres formula (1) when prices are declining, thus creating higher rates of volume growth. The farther away the base year from the current period is, the larger the distortion is. Similarly, price indexes tend to be overestimated for the current year. For this reason, in order to reduce the bias created by the base year, most countries change the base year every five years.

For illustration, table 4 shows changes in rates of volume growth when the base year changes. The indexes for the years prior to the base year are not calculated because one only wants to link the indexes of these series together instead of having a new set of rates of changes for the years prior to the base year, which is similar to the rewriting of history. (The linking of series with different base years will be explained later.) As the base year is closer to the current year, rates of growth are reduced. This is true for both the Laspeyres and Fisher indexes. For example in table 4, annual rate of growth between year 15 and 20 calculated by the Fisher index is 15.9 per cent when base year 0 is used, 15.4 per cent when base year 10 is used, and 13.7 per cent when base year 15 is used. Table 4 also shows that rates of growth by Fisher indexes are normally lower than those of Fisher indexes, except year 10 when substitution effects do not take place as previously explained.

Table 4. Effects by base year changes: rates of volume growth over previous year (%)

Year	**Fisher index**			**Laspyeres index**		
	Base year 0	Base year 10	Base year 15	Base year 0	Base year 10	Base year 15
10	29.3			29.2	29.4	31.2
15	90.5	88.3		102.4	100.5	76.8
20	15.9	15.4	13.7	23.5	22.9	13.8

Data sources: see table A, appendix.

Changing the base year is like trying to rewrite history, as rates of changes in volume and prices change. A base year from the very far-away past will create unrealistically higher rate of changes for the recent years. In the USA, the 1987 fixed weights increase real GDP annual growth rate from 2.3 percent to 2.4 percent. On the contrary, a more recent base year will underestimate the rate of changes in the past. In the USA, with the 1987 fixed weights, real GDP annual growth for 1929-87 reduces from 3.4 percent to 3 percent and especially real GDP dropped 25 percent from 1944 to 1947, but with more appropriate weights, GDP declined by only 13 percent. (See ref. 3). Linking series with different base years is a way to avoid the rewriting of history.

2. Linking series of different base years

Laspeyres volume indexes need to change the base year when the current year is too far away from the base year in order to reduce the rate of growth created by changes in relative prices and their consequent substitution effects. Whenever that happens, it is sometimes necessary to link (or chain) the two data series for analysis so as not to change the rates of growth in the past. The result is that GDP at constant values for previous series is not equal to the sum of the components at the new base constant prices.

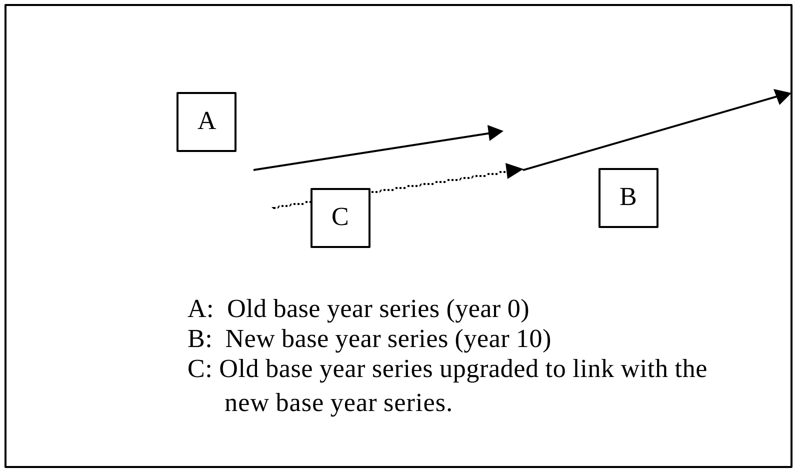
Linking can be simply done by extrapolating backward the index of the new base year using the rates of change in the indexes of the old base year, assuming that the new base year is used as the *reference year* (which is normally the case). Figure 2 shows the linking procedure. The total values of the old series are calculated by using the new rebased indexes. The components of the old series which move with their own quantities and prices must be individually calculated using prices of the new base year in order not to destroy the structure of the expenditures. This linking creates *additvity problem* like the Fisher index: the re-based components do not add up to the new total. In table 5, as an example, for year 0, after linking, the sum of the components is 45 + 19 = 64, which is not the same as 64.07. If year 0 is used as the reference year, the index series of new base year 10 must be grossed down to the reference year.

Table 5: Example of linking data of the two base years

	Base: year 0		**Base: year 10**		
	Year 0	Year 10	Year 10	Year 15	Year 20
	Q_0P_0	$Q_{10}P_0$	$Q_{10}P_{10}$	$Q_{15}P_{10}$	$Q_{20}P_{10}$
Non-high-tech	45	60	60	90	90
High-tech	20	24	22.8	76	114
Total	65	84	82.8	166	204
Composite volume index	1.0000	1.2923	1.0000	2.0048	2.4638
	After linking to base year 10				
Non-high-tech	45		60	90	90
High-tech	19		22.8	76	114
Total	64.07		82.8	166	204
Composite volume index	0.7738		1.0000	2.0048	2.4638

Data sources: see table A, appendix.

Figure 2: Linking procedure



The change of base year and the linking of two or many series with different base years is widely practiced. Rates of growth are kept the same as they are with the new base. The concept of linking is therefore different from the concept of recalculating indexes for previous years using the new base year. The latter is the rewriting of history as growth rates change. However, linking creates additivity problem.

3. Chaining/linking annually rebased indexes

Chaining is nothing else than linking, conceptually. Chaining is the term used when the base year changes annually. An example in table 6 below shows the chaining of annual Fisher indexes. The Laspeyres, Paasche or Fisher indexes must be calculated for every two consecutive years. Again to clarify the issue, both the direct Fisher and chain volume indexes are shown.

The direct Fisher index: This index is calculated to measure change between two years by taking into account prices and quantities of any two years of the 20 year period only (see formulas 1, 2, and 3). A direct volume index compares a terminal year, say year 20, to an initial year, say year 0. The index from year 0 to year 20 is 2.8543.

The linked (chain) Fisher index: The Fisher volume indexes, the series: 1.2930, 1.8829, 1.1266 in bold face in table 6, are obtained by changing the base year annually. These annually linked indexes are now widely called the chain Fisher index. The linking or chaining of these indexes into a time series is simple. One can pick any year as the *reference year* with an index of 100, indexes of other years are then scaled to that year using the annual percentage change provided by the annual indexes. For example, Chain volume index (year 0 = 100)

Year 0	100
Year 10	129.3
Year 15	243.5
Year 20	274.3

Given the current value at year 0, it is possible to calculate the values at constant prices of other years by multiplying the value at year 0 with the above volume indexes. Using the chain index as given above, the chain index for year 0 to 20 = F(0,10) * F(10,15) * F(15,20) = 1.2930 * 1.8829 * 1.1266 = 2.7428. This chain index is smaller than the index 2.8543 between two years (base year 0) shown in table 6, column 2.

Table 6. Fisher's direct index and chain volume index

Terminal year	Initial year			
	Year 0	Year 10	Year 15	Year 20
Year 0	1.0000			
Year 10	**1.2930**	1.0000		
Year 15	2.4633	**1.8829**	1.0000	
Year 20	2.8543	2.1729	**1.1266**	1.0000

Data sources: see table A, appendix.

III. Indexes in national accounts

From the discussions above, theoretically the same annual data on prices and quantities are used for the calculation of both Laspeyres or Fisher indexes. So there is no reason why Fisher chain indexes cannot be calculated, even though it creates the problem of additivity. Laspeyres also has an additity problem when series of different base years are linked. The use of the Fisher indexes as recommended by the SNA will provide analysts with more reliable rates of volume growth and price changes. The Fisher indexes do not require more information.

Practically, detailed data on quantity and price may not be available for many commodities. And unless the commodities are detailed and homogeneous, data on quantity may be misleading. For example, the number of 100 TVs is misleading if it includes TVs of different sizes, HDTV and normal TV, black and white and color. Normally values are collected but not quantities; prices and quantities may be available for almost all goods but not services. For many services it is simply not possible to define quantities objectively. Statisticians may have to use some proxies to measure quantity such as number of patients for quantity of medical services, employment for government services, etc. In order to facilitate different ways of exploiting data in different forms, the formulas below are the results of the reformulation of formulas (1) and (3) used in calculating Laspeyres and Paasche indexes. Given that these indexes are available, Fisher indexes follow suit by using formula (5). Below, two adjacent periods are used in the notation: period 0 and period 1. v_0 is the value at current prices at period 0, v_1 is the value at current prices at period 1. The derivation of formulas(1') and (1'') is shown in the footnotes. Formulas (3') and (3'') are similarly derived.

Lasperes volume index (base year 0)

(1') $L_{q1}(p_0) = (\sum v_0 * (q_1/q_0) / \sum v_0$ [6]

(1") $L_{q1}(p_0) = (\sum v_1 / (p_1/p_0) / \sum v_0$ [7]

Paasche volume index (base year 1)

(3') $Q_{q1}(p_1) = (\sum v_1 / \sum v_1 / (q_1/q_0)$

(3") $Q_{q1}(p_1) = (\sum v_1 / \sum v_0 * (p_1/p_0)$

Formula (1') shows that the Laspeyres volume index of period 1 is obtained by first obtaining the value of period 1 at the prices of period 0. This can be done by extrapolating the current value of period 0 by a proxy of quantity index q_1/q_0. Formula (1") shows that the same thing can be obtained by first deflating the current value of period 1. Similarly, formula (3') shows that the Paasche volume index of period 1 can be obtained by first obtaining the value of period 0 at the prices of period 1 by extrapolating backward the value of period 1 by a proxy of quantity index q_1/q_0.[8] Formula (3") shows that the same thing can be obtained by first inflating the value of period 0 to the prices of period 1. The above formulas allow statisticians to calculate volume indexes without resorting to direct statistics on quantities. Formulas (1'), (1'"), (3') and (3") rely mainly on current values and price statistics that are more readily available. Assuming that one needs to calculate the Laspeyres and Paasche volume indexes of health care but knowing only the revenues and the number of patients for the current and previous periods, one may use formulas (1') and (3') using the number of patients as proxy for the change in quantity of health care. Given Laspeyres and Paasche indexes, Fisher indexes can be easily calculated. Formulas (1") and (3") rely on deflating and inflating techniques given price indexes are available, which may be obtained more easily then quantity indexes. Price indexes for individual and detailed products are important for the estimation of group indexes. The homogeneity requirement in a product is important in price statistics, but not as important as in the quantity of a product. Normally when no statistics on sales of individual products are available, for example butter, which may consist of different kind, a simple un-weighted average or the price index of the most representative one may be acceptable. (See ref. 1, 8). On the basis of the above formulas, it is easier to understand the data requirement for obtaining GDP and value added at constant prices.

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[6] The Laspeyres quantity index can be rewritten as follows:

$$L_{q1}(p_0) = \sum p_0 q_1 / \sum p_0 q_0 = (\sum p_0 q_0 * (q_1/q_0)) / \sum p_0 q_0 = (\sum v_0 * (q_1/q_0) / \sum v_0$$

Where v_0 is the base year value, q_t/q_0 can be replaced by any proxy index if quantities of period t are not available.

[7] Another of writing the Laspeyres quantity index is as follows:

$$L_{q1}(p_0) = (\sum (p_0\ p_1 q_t / p_{1)}) / \sum p_0 q_0 = (\sum v_1 / (p_1/p_0) / \sum v_0$$

Where v_1 is deflated and then divided by v_0 to get the quantity index.

[8] When proxy for quantity is used, the resulting index ignores change in quality. Quality, which is notoriously difficult to estimate, has to be introduced into the index by a separate study. See ref. 2, 7, 8.

1. GDP/Value added

GDP can be estimated by three methods: (1) the production approach; (2) the income approach; and (3) the final expenditure approach.

The production approach obtains the value added for each industry by deducting intermediate consumption from output and then summing up the value added to obtain GDP. To apply Laspeyres and especially Fisher formulas appropriately, one needs output of each industry by type of products and also intermediate consumption by type of products consumed by each industry for every year. To arrive at GDP at constant prices, one needs to get output at constant prices and intermediate consumption at constant prices in order to get value added by industry. (This is the double deflation method[9]). Data requirement for this task are hardly met by any country and more so in developing countries, especially at the end of the current year. Some proxies or quantity indexes have to be created for the more current years as previously discussed. Deflating, using this approach, provides not only GDP but also value added by industry at constant prices. This approach is the most data demanding. Many developing countries simply use the consumer price index (CPI) to deflate GDP. Price indexes required are of the producer or basic price indexes depending on whether taxes on products are included or excluded from unit prices collected.

The final expenditures approach sums up final consumption expenditures of households, government and non-profit institutions, gross capital formation, exports less imports to obtain GDP. This approach provides a less demanding way to calculate volume index for GDP because only final goods and services are required. Formulas of the type (1)-(5) are used. The final expenditure approach is a practice in the United Kingdom, the USA, Canada and other countries. Consumer price indexes are be used for final consumption expenditure, producer price indexes used for gross capital formation. Export and import price indexes are also required.

The income approach sums up components of value added to obtain GDP. Constant GDP has never been obtained by using this approach since it is theoretically impossible to deflate operating surplus or profit.

2. CPI

Overall consumer price index (CPI) is generated by dividing the values at current prices by the base year prices of the same basket of goods and services purchased by households at the base year. It is an index widely used to determine wages in labor contracts, income transfer programs of government, etc. This is a Laspeyres price index. Because of that, technically it cannot be used to deflate current values; only the Paasche price index should be used (see item a in section 3, part I.). With current weights, the Paashce CPI index can be developed. It reflects better the substitution effects and therefore is preferred by economists. It should also be mentioned that CPI is mostly developed for urban areas on the basis of the basket of goods and services purchased by urban residents. It is not fully

[9] The double deflation method is discussed in depth in the *Handbook of Input-Output Table Compilation and Analysis* (see ref. 6).

compatible with the SNA concept of final consumption expenditures of households, as there are goods and services produced for own final consumption and other imputed expenditures incorporated by the SNA such as own-occupied housing, etc that are not included in the basket of goods and services. To broaden the CPI concept to rural areas and to make it compatible with the SNA is more demanding, more data are required since production for own final consumption in rural areas is much higher than in urban areas.

Appendix

Table A. The example used in the paper

	Year 0			Year 10			Year 15			Year 20		
	Quantity	Price	Value	Quantity	Price	Value	Quantity	Price	Value	Quantity	Price	Value
Non-high-tech	15	3	45	20	3	60	30	3.5	105	30	4	120
High-tech	5	4	20	6	3.8	22.8	20	2	40	30	1.8	54
Total			65			82.8			145			174
Structure												
Non high-tech			0.692			0.725			0.724			0.690
High-tech			0.308			0.275			0.276			0.310

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